Family of Ladder Graphs are Properly Lucky

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Abstract

The proper labeling is different natural number for adjacent nodes. Lucky labeling is that the total of labels over adjacent nodes within the graph is not same. Lucky labeling is linked with proper labeling. The aim of the paper is to show proper lucky labeling and proper lucky number of a ladder graph, open ladder graph, slanting ladder graph, triangular ladder graph, open triangular graph, diagonal ladder graph and open diagonal ladder graph.

Keywords: Ladder Graph, Open Ladder Graph, Slanting Ladder Graph, Triangular Ladder Graph, Open Triangular Graph, Diagonal Ladder Graph, Open Diagonal Ladder Graph.

INTRODUCTION

Graph labeling is a broad in graph theory and more innovative results in past few decades. Most researchers have initiated several graph labelling in recent decades. The proper labeling is different natural number for adjacent nodes. Lucky labeling is that the total of labels over adjacent nodes within the graph is not same [1]. Lucky and proper labeling was linked. It is represented by $\eta_p(G)$ [2]. And labelling and sum over neighbor nodes is represented by $f$ and $S$ respectively.

Distance irregular, quotient, logarithmic mean, prime, geometric mean, tri magic and sum divisor labeling of ladder graph was derived by researchers. Ladder graphs applications are digital to analog conversion, electrical areas and wireless communication area such as Wi-Fi, cellular phones etc., Our aim was to compute the $\eta_p(G)$ of a ladder graph, open ladder graph, slanting ladder graph, triangular ladder graph, open triangular graph, diagonal ladder graph and open diagonal ladder graph.

PRELIMINARIES

A. Ladder Graph

The ladder graph has vertices $a_i$ and $b_j$ are the two paths in the graph $V(G) = \{a_i, b_j : i = j, 1 < i \leq n, 1 < j \leq n\}$ and the edge of are $E(G) = \{a_i_a_{i+1}, b_jb_{j+1} : i = j, 1 \leq i \leq n, 1 \leq j \leq n\} \cup \{a_i, b_j : i = j, 1 \leq i \leq n, 1 \leq j \leq n\}$ refer figure 1. It is denoted by $L_n$ [3].

B. Open Ladder Graph

An Open ladder is generated from a ladder graph with $n > 2$ by excluding the edges $a_i b_j$, for $i = 1$ and $n, j = 1$ and $n$ refer figure 2. It is denoted by $OL_n$ [3].

C. Slanting Ladder Graph

A slanting ladder is the graph obtained from two paths $a_i$ and $b_j$ by joining each $a_i$ with $b_{j+1}$, $1 \leq i \leq n - 1, 1 \leq j \leq n - 1$ refer figure 3. It is denoted by $SL_n$ [3].
Theorem 3.1
The Ladder graph $L_n$ for $n > 1$ is properly lucky with $\eta_p(L_n) = 2$.

$\eta_p(L_n) = 2$

Figure 8: proper lucky ladder graph $L_5$
**Illustration 3.2:** The proper lucky labeling of ladder graph with \( n = 5 \) is shown in figure 8.

**Theorem 3.3**
The Open Ladder graph \( OL_n \) for \( n > 2 \) is proper lucky with \( \eta_p(OL_n) = 2 \).

**Proof**
Let \( f: V(G) \to \{1,2\} \) for open ladder graph \( OL_n \) for \( n > 2 \) be defined by,

\[
\begin{align*}
f(a_i) &= \begin{cases} 
1 & \text{odd } i \\
2 & \text{even } i
\end{cases} \\
f(b_j) &= \begin{cases} 
1 & \text{odd } j \\
2 & \text{even } j
\end{cases}
\end{align*}
\]

\[s(a_1) = 2, \quad s(b_1) = 1\]

\[s(a_n) = \begin{cases} 
1 & \text{even } n \\
2 & \text{odd } n
\end{cases}, \quad s(b_n) = \begin{cases} 
1 & \text{odd } n \\
2 & \text{even } n
\end{cases},
\]

\[s(a_i) = \begin{cases} 
3 & \text{even } i < n \\
6 & \text{odd } i \text{ and } 3 \leq i < n
\end{cases}, \quad s(b_j) = \begin{cases} 
3 & \text{even } j < n \\
6 & \text{odd } j \text{ and } 3 \leq j < n
\end{cases}\]

The minimum value of \( V(G) \) is 2. Therefore it is proper lucky with \( \eta_p(OL_n) = 2 \).

**Illustration 3.4:** The proper lucky labeling of open ladder graph \( OL_5 \) is shown in figure 9.

**Theorem 3.5**
The Slanting Ladder graph \( SL_n \) for \( n > 1 \) is proper lucky with \( \eta_p(SL_n) = 2 \).

**Proof**
Let \( f: V(G) \to \{1,2\} \) for slanting ladder graph \( SL_n \) for \( n > 1 \) be defined by

\[
\begin{align*}
f(a_i) &= \begin{cases} 
1 & i = 3k - 1 \\
2 & i = 3k - 2 \\
3 & i = 3k
\end{cases} \\
f(b_j) &= \begin{cases} 
2 & j = 3k - 1 \\
3 & j = 3k - 2
\end{cases}
\end{align*}
\]

\[s(a_1) = 4, \quad s(b_1) = 5\]

\[s(a_n) = \begin{cases} 
4 & n = 3k \\
7 & n \neq 3k
\end{cases}\]

**Illustration 3.6:** The proper lucky labeling of slanting ladder graph with \( n = 6 \) is shown in figure 10.

**Theorem 3.7**
The Triangular Ladder graph \( TL_n \) for \( n > 1 \) is proper lucky with \( \eta_p(TL_n) = 3 \).

**Proof**
Let \( f: V(G) \to \{1,2,3\} \) for triangular ladder graph \( TL_n \) for \( n > 1 \) be defined by

\[
\begin{align*}
f(a_i) &= \begin{cases} 
1 & i = 3k - 1 \\
2 & i = 3k - 2 \\
3 & i = 3k
\end{cases} \\
f(b_j) &= \begin{cases} 
2 & j = 3k - 1 \\
3 & j = 3k - 2
\end{cases}
\end{align*}
\]

\[s(a_1) = 4, \quad s(b_1) = 5\]

\[s(a_n) = \begin{cases} 
4 & n = 3k \\
7 & n \neq 3k
\end{cases}\]
\( s(b_n) = \begin{cases} 
3 & n = 3k - 2 \\
4 & n = 3k - 1 \\
5 & n = 3k 
\end{cases} \)

\( s(a_i) = \begin{cases} 
6 & i = 3k \text{ and } i < n \\
8 & i = 3k - 2 \text{ and } 4 \leq i < n \\
10 & i = 3k - 1 \text{ and } i < n 
\end{cases} \)

\( s(b_j) = \begin{cases} 
6 & j = 3k - 2 \text{ and } 4 \leq j < n \\
8 & j = 3k \text{ and } j < n \\
10 & j = 3k - 1 \text{ and } j < n 
\end{cases} \)

The minimum value of \( V(G) \) is 3. Therefore it is properly lucky with \( \eta_p(TL_n) = 3 \).

\[ \begin{align*}
2(4) & \quad 1(10) & \quad 3(6) & \quad 2(8) & \quad 1(7) \\
3(5) & \quad 2(8) & \quad 1(10) & \quad 3(6) & \quad 2(4)
\end{align*} \]

Figure 11: Proper lucky triangular ladder graph \( TL_n \)

**Illustration 3.8:** The proper lucky labeling of triangular ladder graph with \( n = 5 \) is shown in figure 11.

**Theorem 3.9**
The Open Triangular Ladder graph \( OTL_n \) for \( n > 2 \) is properly lucky with \( \eta_p(OTL_n) = 3 \).

**Proof**
Let \( f : V(G) \rightarrow \{1,2,3,4\} \) for open ladder triangular graph \( OTL_n \) for \( n > 2 \) be defined by

\[ f(a_i) = \begin{cases} 
1 & i = 3k - 2 \\
2 & i = 3k - 1 \\
3 & i = 3k 
\end{cases} \]

\[ f(b_j) = \begin{cases} 
1 & j = 3k - 1 \\
2 & j = 3k \\
3 & j = 3k - 2 
\end{cases} \]

\[ s(a_1) = 2 \\
\]

\[ s(b_1) = 3 \\
\]

\[ s(a_n) = \begin{cases} 
3 & n = 3k \\
4 & n = 3k + 2 \\
5 & n = 3k + 1 \\
1 & n = 3k 
\end{cases} \]

\[ s(b_n) = \begin{cases} 
2 & n = 3k + 1 \\
3 & n = 3k + 2 
\end{cases} \]

\[ s(a_i) = \begin{cases} 
6 & i = 3k \text{ and } i < n \\
8 & i = 3k - 1 \text{ and } i < n \\
10 & i = 3k - 2 \text{ and } 4 \leq i < n 
\end{cases} \]

\[ s(b_j) = \begin{cases} 
8 & j = 3k \text{ and } j < n \\
10 & j = 3k - 1 \text{ and } j < n 
\end{cases} \]

The minimum value of \( V(G) \) is 3. Therefore it is properly lucky with \( \eta_p(OTL_n) = 3 \).

\[ \begin{align*}
1(2) & \quad 2(8) & \quad 3(6) & \quad 1(10) & \quad 2(4) \\
3(3) & \quad 1(10) & \quad 2(8) & \quad 3(6) & \quad 1(3)
\end{align*} \]

Figure 12: Proper lucky open triangular graph \( OTL_n \)

**Illustration 3.10:** The proper lucky labeling of open triangular ladder graph with \( n = 5 \) is shown in figure 12.

**Theorem 3.11**
The Diagonal graph \( DL_n \) for \( n > 1 \) is properly lucky with \( \eta_p(DL_n) = 4 \).

**Proof**
Let \( f : V(G) \rightarrow \{1,2,3,4\} \) for diagonal ladder graph \( DL_n \) for \( n > 1 \) be defined by

\[ f(a_i) = \begin{cases} 
1 & \text{odd } i \\
2 & \text{even } i 
\end{cases} \]

\[ f(b_j) = \begin{cases} 
3 & \text{odd } j \\
4 & \text{even } j 
\end{cases} \]

\[ s(a_1) = 9 \\
\]

\[ s(b_1) = 7 \\
\]

\[ s(a_n) = \begin{cases} 
8 & \text{even } n \\
9 & \text{odd } n \\
10 & \text{even } n 
\end{cases} \]

\[ s(b_n) = \begin{cases} 
6 & \text{odd } n \\
7 & \text{even } n \\
12 & \text{even } i \text{ and } 2 \leq i < n \\
15 & \text{odd } i \text{ and } 3 \leq i < n \\
13 & \text{even } j \text{ and } 2 \leq j < n \\
10 & \text{odd } j \text{ and } 3 \leq j < n 
\end{cases} \]

The minimum value of \( V(G) \) is 3. Therefore it is properly lucky with \( \eta_p(DL_n) = 4 \).

\[ \begin{align*}
1(9) & \quad 2(12) & \quad 1(15) & \quad 2(12) & \quad 1(9) \\
3(7) & \quad 4(10) & \quad 3(13) & \quad 4(10) & \quad 3(7)
\end{align*} \]

Figure 13: Proper lucky diagonal ladder graph \( DL_n \)
The proper lucky labeling of diagonal ladder graph with \( n = 5 \) is shown in figure 13.

**Theorem 3.13**

The Open Diagonal graph \( ODL_n \) for \( n > 2 \) is proper lucky with \( \eta_p(ODL_n) = 4 \).

**Proof**

Let \( f : V(G) \to \{1,2,3,4\} \) for open diagonal ladder graph \( ODL_n \) for \( n > 2 \) be defined by

\[
\begin{align*}
f(a_i) &= \begin{cases} 1 & \text{even } i \\ 2 & \text{odd } i \end{cases} \\
f(b_j) &= \begin{cases} 3 & \text{even } j \\ 4 & \text{odd } j \text{ and } j > 1 \end{cases}
\end{align*}
\]

The minimum value of \( V(G) \) is 3. Therefore it is proper lucky with \( \eta_p(ODL_n) = 4 \).

The proper lucky labeling of open diagonal ladder graph \( ODL_5 \) is shown in figure 13.

**Illustration 3.14:** The proper lucky labeling of open diagonal ladder graph with \( n = 5 \) is shown in figure 14.

**Conclusion**

In this article, we found proper lucky labeling for ladder graph, open ladder graph, slanting ladder graph, triangular ladder graph, open triangular ladder graph, diagonal ladder graph and open diagonal ladder. We obtained

\[
\begin{align*}
\eta_p(L_n) &= \eta_p(OL_n) = \eta_p(SL_n) = 2 \\
\eta_p(TL_n) &= \eta_p(OTL_n) = 3
\end{align*}
\]

Further, we are do this to various graphs like triangular family.

**References**

