

# MAGNETIC FIELD INFLUENCE ON RADIATIVE CASSON FLUID FLOW OVER AN EXPONENTIAL STRETCHING SURFACE

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## Abstract

The flow and heat transfer properties of a Casson liquid past an exponential stretching sheet under the influence of transverse magnetic-field are investigated in this work. The numerical simulations were solved by using Keller Box method. The impact of various flow quantities such as the Casson fluid parameter, magnetic-field parameter, Prandtl number, and suction parameters on velocity and temperature fields are analysed through graphs. Moreover, the current method is compared with the existing methods, and the results are found to be similar.

**Keywords:** Casson fluid; Transverse magnetic field; Radiative heat flux; Exponential sheet; Keller – Box- Method.

## 1. INTRODUCTION

It is significant to memorize here that many industrially relevant fluids are non-Newtonian. Non-Newtonian type fluids are now generally recognised as vital role in engineering and industrial applications such as petroleum exploration, polymer design, separation processes, food and paper manufacturing units, and so on. (Casson 1959; Cebeci Bradshaw 1988).

Heat transport of a second-grade fluid via a porous media with the effect of chemical reaction over a linear stretched surface has been investigated by (Chandra Sekhar 2018). (Konda. Madhusudhana and Konijeti 2018) investigated the MHD Casson nanofluid flow across a nonlinear stretching surface with suction/injection and heat transfer effects. The effects of heat and mass transfer on MHD have been numerically investigated by Talla H., Kumari P., Sridhar W. (2018). In this study, the authors considered Casson fluid model for developing governing equations past an exponential sheet. (Vijaya and Reddy 2019) examined the effects of thermal diffusion and destructive type chemical reactions on radiative Casson fluid flow across a semi-infinite vertical plate embedded in a porous material. Using the Casson model between two rectangular plates, (Sampath Kumar and Pai 2019) investigated the effects of suction and injection on incompressible fluid flow. (Kumar, Reddy and Raju 2019) proposed a solution for natural convection MHD flow of viscous and heat-absorbing fluid over an infinite vertical porous plate in the presence of thermal diffusion. (Manjula and Chandra Sekhar 2019) studied the effects of Soret and heat generation on MHD flow using the Casson model over an inclined porous plate. (Raghunandana Sai and Ramana Murthy 2019) examined a free convection in a compressible and electrically steered thick liquid along an unbounded permeable medium.

The authors (Suneetha, Ibrahim and Ramana Reddy 2019) investigated under the impact of heat and mass transfer effects on unsteady convention flow of viscous fluid embedded in a high porous medium.in this investigation the authors considered an infinite vertical moving plate. (Sivaiah et.al. 2019) explored the flow of visco-elastic and dissipative fluid in the presence of thermal radiation and variable permeability. (Nagalakshmi and Vijaya 2020) The Carreau nano liquid-boundary layer flow and heat transmission over a non-linear stretching surface were investigated by (Nagalakshmi and Vijaya 2020) investigated the free convection flow of a conducting nanofluid bounded by a flat plate in the presence of heat and mass transfer effects. The numerical investigation of an MHD fluid flow past a stretching porous surface with the effect of destructive chemical reaction was studied by (Hymavathi and Sridhar 2016). In this examination the boundary layer dimensional equations was solved by using Keller box technic.

The numerical solution for the MHD Casson liquid flow with thermal radiation past an exponential stretching surface is investigated. The Keller Box method is used for solving non dimensional equations. Further the obtained results are presented and analyzed through graphs. The impact of various flow quantities on velocity and temperature fields is discussed.

## 2. Formulation of the Problem

The two-dimensional boundary layer flow of a Casson fluid across a flat surface in the presence of radiative heat flux is examined. A perpendicular to the flow direction transverse magnetic field is applied. The x-axis is seen as a flow direction. The wall is stretched while the origin is kept stationary by applying equal and opposite forces along the x axis. (see Fig. 1). The rheological equations of state for an incompressible isotropic Casson fluid are as follows

$$\tau_{ij} = \begin{cases} 2(\mu_B + P_y/\sqrt{2\pi})e_{ij}, \pi > \pi_c \\ 2(\mu_B + P_y/\sqrt{2\pi_c})e_{ij}, \pi < \pi_c \end{cases} \quad (1)$$

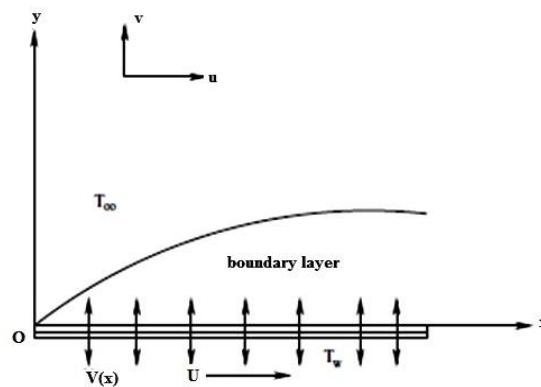


Fig.1 Schematic diagram

Here  $\pi = e_{ij}e_{ij}$  and  $e_{ij}$  is the deformation and product of deformation components are denoted by  $(i,j)$ th and  $\pi$  respectively.  $\pi_c$ ,  $\mu_B$  and  $P_y$  denotes the critical value, plastic viscosity and yield stress of the fluid.

The governing flow equations under the above boundary layer approximations are

$$u_x + v_y = 0 \quad (2)$$

$$u u_x + v u_y = \nu \left( 1 + \frac{1}{\beta} \right) u_{yy} - \frac{\sigma B_0^2}{\rho} u \quad (3)$$

$$u T_x + v T_y = \frac{K}{\rho C_p} T_{yy} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} \quad (4)$$

Where,  $(u,v)$  are the velocity components along  $(x,y)$  directions respectively.  $\nu$  the kinematic viscosity,  $\rho$  is the fluid density,  $\beta = \mu_B \sqrt{2\pi_c} / P_y$ , the Casson parameter,  $\sigma$  is electrical conductivity of the fluid,  $K$  is the thermal diffusion coefficient,  $q_r$  is radiative heat flux and  $C_p$  is the specific heat capacitance.

The Rosseland approximation

$$q_r = -\frac{4\sigma}{3k^*} \frac{\partial T^4}{\partial y} \quad (5)$$

Here  $\sigma$  is the Stefan-Boltzman constant and  $k^*$  is the absorption coefficient.

$$\text{The Taylor series expansion of } T^4 \text{ about } T_\infty \text{ is considered as } T^4 \equiv 4T_\infty^3 T - 3T_\infty^4 \quad (6)$$

In view of equation (6), the equation (4) becomes

$$uT_x + vT_y = \frac{K}{\rho C_p} T_{yy} + \frac{16\sigma T_\infty^3}{3\rho C_p k^*} T_{yy} \quad (7)$$

The associated boundary conditions are

$$u = U, v = -V(x), T = T_w \text{ at } y = 0 \quad (8)$$

$$\text{as } y \rightarrow \infty, u \rightarrow 0, T \rightarrow T_\infty \quad (9)$$

Here  $U = U_0 e^{\frac{x}{2L}}$  is the stretching velocity,  $T_w = T_\infty + T_0 e^{\frac{x}{2L}}$  is the temperature at the sheet,  $U_0$  and  $T_0$  are the reference velocity and temperature respectively.  $V(x) = V_0 e^{\frac{x}{2L}}$  is a special type of velocity at the wall with  $V_0$  as constant.  $V(x) > 0$  is the velocity suction and  $V(x) < 0$  is the velocity blowing.

### 3. SOLUTION OF THE PROBLEM

Introducing the following similarity transformations

$$\eta = \sqrt{\frac{U_0}{2vL}} e^{\frac{x}{2L}} y, U = U_0 e^{\frac{x}{2L}} f'(\eta)$$

$$v = -\sqrt{\frac{vU_0}{2L}} e^{\frac{x}{2L}} [f(\eta) + \eta f'(\eta)]$$

$$T = T_\infty + T_0 e^{\frac{x}{2L}} \theta(\eta) \quad (10)$$

Under the above similarity transformations, the governing dimensional equations (3) and (4) & the corresponding boundary conditions reduces to

$$\left(1 + \frac{1}{\beta}\right) f''' + f f'' - 2f'^2 - M f' = 0 \quad (11)$$

$$\left(1 + \frac{4N}{3}\right) \theta'' + Pr(f\theta' - \theta f') = 0 \quad (12)$$

$$\text{at } \eta = 0, f(\eta) = S, \theta(\eta) = 1, f'(\eta) = 1 \quad (13)$$

$$\text{as } \eta \rightarrow \infty, f'(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0 \quad (14)$$

$$\text{Where } S = v_0 / \sqrt{\frac{U_0 v}{2L}}, M = \frac{2\sigma B_0^2 l}{\rho U_0 e^{x/l}}, N = \frac{4\sigma T_\infty^3}{k k^*} \text{ and } Pr = \frac{\mu C_p}{k}.$$

### 4. RESULTS AND DISCUSSION

The Keller box approach, which includes the finite difference technique, Newton's scheme, and block elimination method, is used to solve the transformed nonlinear equations (8) and (9) with their associated conditions (10) via MATLAB. The obtained graphical results for different parameters such as Casson, Magnetic, Suction parameters, and Prandtl number on velocity and

temperature fields are discussed. The present results using Keller Box approach are compared with the results of ( Magyari and Keller 1999; Pramanik 2014 ). Tables 1 and 2 demonstrate the comparisons, which are extremely close.

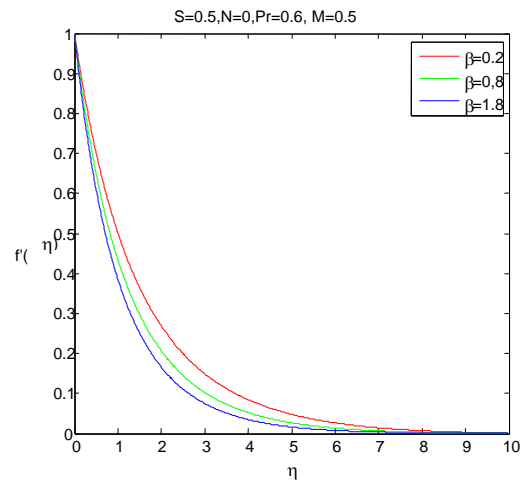


Fig.2. Effect of  $\beta$  on  $f'(\eta)$  .

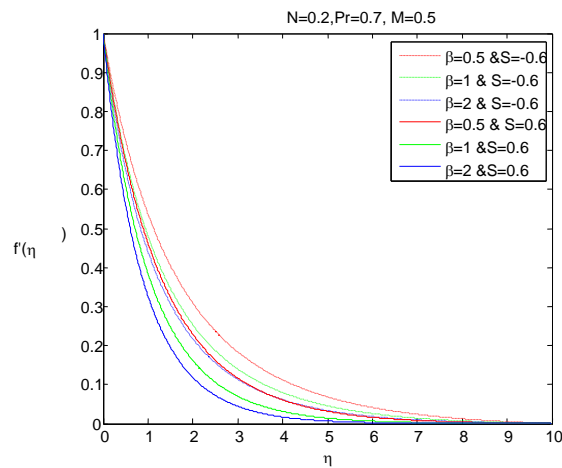


Fig.3 Effect of  $\beta$  on  $f'(\eta)$  for suction/blowing and  $N=0.2$ .

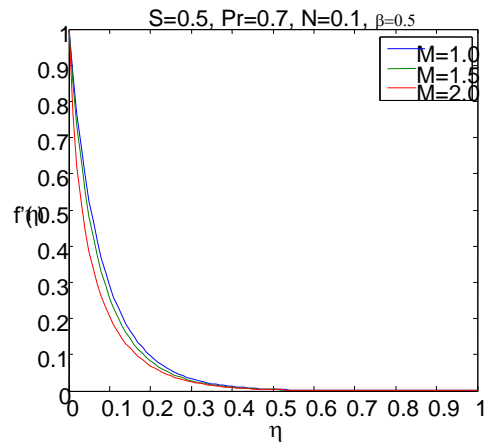


Fig.4 Effect of  $M$  on  $f'(\eta)$

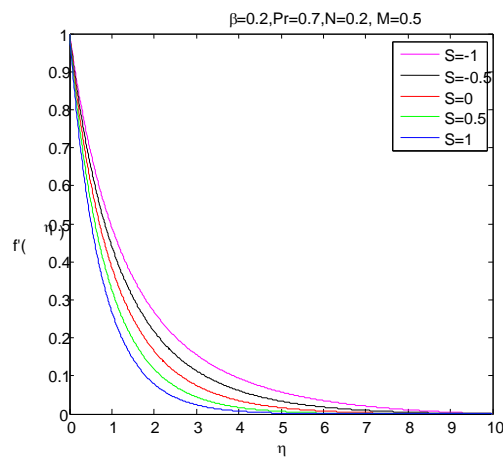


Fig.5 Effect of  $S$  on  $f'(\eta)$

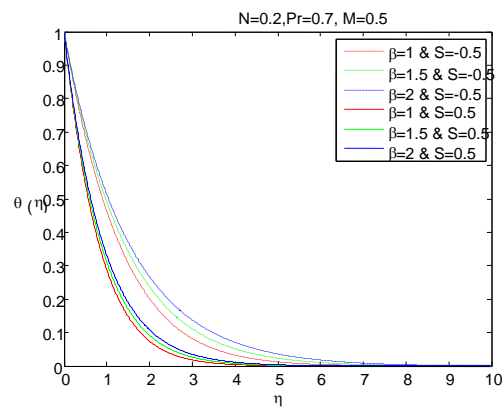


Fig.6 Effect of  $\beta$  on  $\theta(\eta)$  in presence of suction/blowing

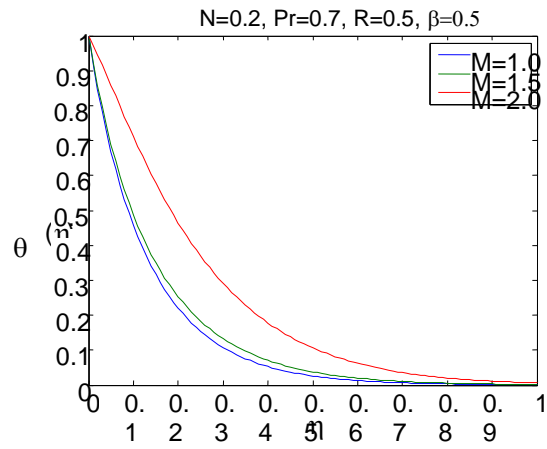


Fig. 7 Effect of  $M$  on  $\theta(\eta)$

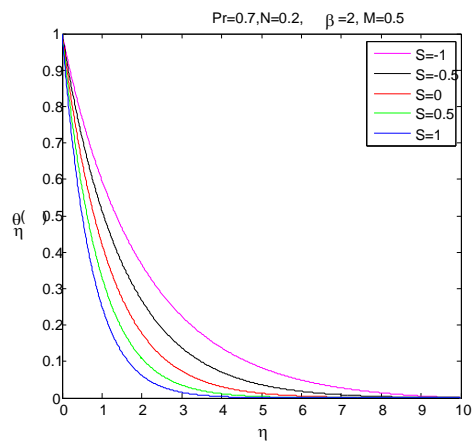


Fig. 8 Effect of  $S$  on  $\theta(\eta)$ .

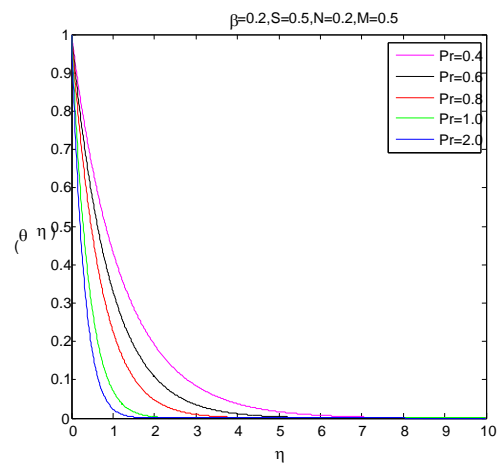


Fig. 9 Effect of  $Pr$  on  $\theta(\eta)$  in presence of suction parameter  $S$

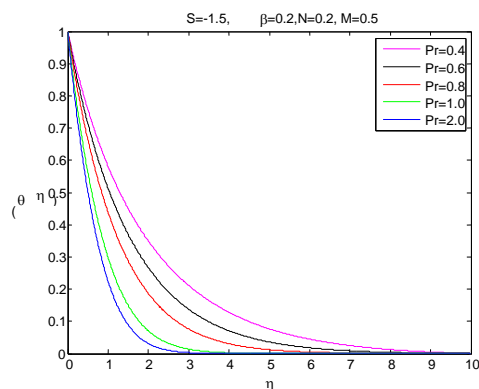


Fig. 10 Effect of  $Pr$  on  $\theta(\eta)$  in presence of blowing parameter  $S$

The velocity profiles for various values of the Casson parameter  $\beta$  in the absence of radiation and in the presence of radiation parameter  $N$  and magnetic parameter  $M$  are depicted in Figs.2-4. The velocity drops with rising values of this parameter in all of these circumstances, which is owing to the fact that the thickness of the momentum boundary layer reduces with rising values of these parameters. In the case of suction, velocity reduces as the suction parameter increases, while the opposite trend is observed for blowing parameter (see Fig. 5). Fig.6 represents the variation in temperature with Casson parameter  $\beta$ . It can be seen that the temperature of the fluid improves with increasing values of the Casson parameter, both during suction and during injection.

This is because of the fact that the higher values of Casson parameter improve the thickness of the thermal boundary layer. Also the fluid temperature is an increasing function of magnetic parameter  $M$  (see Fig.7). The temperature profiles for suction/injection parameter  $S$  is presented in Fig.8. From the figure, it is seen that the fluid temperature depreciates with raising values of  $S$ . Figures 9 - 10 represent the temperature profiles for diverse values of Prandtl number for suction and injection cases.

With higher values of the suction and injection parameters, the thickness of the thermal boundary layer and the temperature of the fluid are observed to decrease. The value of the skin friction for Newtonian fluid ( $\beta \rightarrow \infty$ ) is 1.281750 in the present study which is compared with (Magyari and Keller 1999) and observed that it is good in agreement.

## 5. FINAL REMARKS

In this article we investigated a non-Newtonian Casson fluid flow with MHD over a stretching surface in the presence of thermal radiation. The non-linear ordinary differential equations are solved by using Keller-Box method. The obtained results are analysed and presented through graphs. The major outcomes are

- i. The Casson parameter reduces the thickness of the momentum boundary layer.
- ii. As the suction and blowing parameters are increased, the fluid velocity temperature profiles drop.
- iii. The fluid temperature rises as the Suction/Blowing parameter rises.
- iv. In both suction and injection scenarios, increasing the Prandtl number reduces the thickness of the thermal boundary layer.

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