Multi-Attribute Decision Making Problem Using OWA Operator

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Abstract

The development of decision-making processes lies at core of the fuzzy set. Multi-attribute (MADM) is the most prevalent method of decision-making is of such situations. A common MADM problem involves analysing a comparison between the collection of alternatives and a selection of decision criteria. This paper provides a central government economic benefit to provinces and municipalities by using MADM situations.

Keywords: MADM, OWA Operator, Membership Function.

INTRODUCTION

Multiple criteria decision making (MCDM) was developed by Benjamin Franklin (1772). MCDM has two various types that are Multi-attribute Decision Making (MADM) and Multi-Objective Decision Making (MODM) [6]. MODM model that investigates situations with a continuous choice space. Mathematical programming problems with many objective functions are a good illustration. Kuhn and Tucker are credited with the first mention of this problem, commonly known as the "vector-maximum" problem. [2]. MADM, on the other hand, concentrates on involving distinct decision regions. There is a fixed set of options for decision making in certain circumstances.

Even though MADM approaches vary greatly, many of them share key characteristics [1]. Numerous problems in the fields of information, finance, engineering, and other disciplines are resolved with OWA. The ordered weighted averaging (OWA) operator (OWAO) [9] is another fascinating aggregating operator that, despite being proposed in 1988, has not been frequently employed in the literature. The OWAO belongs to a group of parameterized aggregate operators that span the range of values high and low. Torra's [5] work includes the development of the weighted OWA (WOWA) operator, and Xu's [8] work includes the hybrid averaging (HA) operator. Both approaches arrived at unification between the OWA and the WA since both concepts were included in the formulation as exceptional circumstances. These models, however, appear to be a partial unification rather than a complete one, as shown in [3], despite being able to unify them, they are unable to analyse whether important these ideas are to the particular issue at hand. For instance, we might wish to give the OWA operator more weight in some situations because we think it's more important, and conversely. [4].

PRELIMINARIES

Ordered Weighted Geometric (OWG) [7]
The OWA: \( \mathbb{R}^q \to \mathbb{R} \), Then

\[
OWA_\omega(\beta_1, \beta_2, ..., \beta_q) = \sum_{l=1}^{n} \mu_l \alpha_l
\]

where \( \alpha_l \) be a \( l^{th} \) highest set of arguments \( \beta_o \) (\( o = 1,2, ..., q \)) the arguments \( \alpha_l \) (\( l = 1,2, ..., q \)) be ordered in descending order: \( \alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_l, \mu = (\mu_1, \mu_2, ..., \mu_q) \) be a the weighting vector (WV) connected to the OWA function, \( \mu_l \geq 0, l = 1,2, ..., q, \sum_{l=1}^{p} \mu_l = 1 \), and \( \mathbb{R} \) will be collection of real numbers.

For example,

For \( \mu = (0.1, 0.5, 0.4, 0.2) \) be a WV of OWA operator and (5, 14, 9, 20),

\[
OWA_{\mu}(5, 14, 9, 20) = 0.1 \times 20 + 0.5 \times 14 + 0.4 \times 9 + 0.2 \times 5 = 13.6
\]
Theorem 2.1 [7]
Let $\mu_l = \frac{1-\alpha}{p} + \alpha, \mu_l = \frac{1-\alpha}{p}, l \neq 1$ and $\alpha \in [0,1]$, then
$$\alpha OWA_{\mu_l}(\alpha_1, \alpha_2, ..., \alpha_q) = OWA_{\mu_l}(\alpha_1, \alpha_2, ..., \alpha_q)$$
If $\alpha = 0$, we get
$$OWA_{\mu_l}(\alpha_1, \alpha_2, ..., \alpha_q) = OWA_{\mu_l}(\alpha_1, \alpha_2, ..., \alpha_q)$$
If $\alpha = 1$, we get
$$OWA_{\mu_l}(\gamma_1, \gamma_2, ..., \gamma_q) = OWA_{\mu_l}(\gamma_1, \gamma_2, ..., \gamma_q)$$

Algorithm for MADM Problem Using OWA
Step 1: In MADM problem, $Z = \{z_1, z_2, z_3, ..., z_p\}$ will be a finite collection of alternatives,
$O = \{o_1, o_2, o_3, ..., o_q\}$ is a collection of attributes and its
weight details is completely unspecified. A decision making calculates the
alternatives $z_i$ with respect to the attribute $o_m$ and then we get the values $b_{l,m}$ $(l = 1,2, ..., q; m = 1,2, ..., p)$. $C = (c_{l,m})_{q \times p}$.

Step 2: Generally, MADM problem have some ways of attributes namely
- Benefit
- Cost
- Fixed
- Deviation
- Interval
- Deviation interval
Each attribute values $c_{l,m}$ in the decision matrix $C = (c_{l,m})_{q \times p}$
using the below formulas:
$$p_{l,m} = \frac{c_{l,m}}{\max(c_{l,m})}, l = 1.2,3, ..., q, p \in L_1$$
$$p_{l,m} = \frac{c_{l,m}}{\max(c_{l,m})}, l = 1,2,3, ..., q, p \in L_2$$
Hereafter we develop the decision matrix to $P = (p_{l,m})_{q \times p}$.

Step 3: The OWA aggregate every attribute value in
$p_{l,m}$ $(m = 1,2, ..., p)$ to the alternatives $z_i$. After get the attribute values by using definition 1.
$$z_i(\omega) = OWA_{\mu_l}(p_{l,1}, p_{l,2}, ..., p_{l,p}) = \sum_{i=1}^{q} \mu_{m} a_{l,m}$$
Since $a_{m}$ are $m^{th}$ highest value of $p_{l,m}$ $(m = 1,2, ..., q), \mu = (\mu_1, \mu_2, ..., \mu_p)$ will be a weighting vector with OWA
operator $\mu_p = 0, \sum_{m=1}^{p} \mu_m = 1$, by using theorem 1 or by the
normal distribution.
$$\mu_m = \frac{e^{-\frac{(m-\sigma_p)^2}{2\sigma_p^2}}}{\sum_{i=1}^{p} e^{-\frac{(m-\sigma_p)^2}{2\sigma_p^2}}}, m = 1,2, ..., p$$
Since $\sigma_p = \frac{1}{2}(1+p), \omega_p = \sqrt{\frac{1}{p} \sum_{l=1}^{p} (l - \sigma_p)^2}$

Step 4: Every alternative rank $z_i$ $(l = 1,2,3, ..., q)$
corresponds to the values of $z_i(\mu)$ in descending order.

**NUMERICAL EXAMPLE**
Utilizing data from the China Industrial Economic Statistical
Yearbook, evaluate the financial advantages of 16 cities and
territories directly under the control of the federal
government (2003). Given a collection of alternatives, choose
one of the following: $Z = \{z_1, z_2, z_3, ..., z_{16}\} = \{Anhui,$
Beijing, Fujian, Guangdong, Hebei, Henan, Hunan, Jiangsu,
Jiangxi, Liaoning, Shandong, Shanghai, Shanxi, Tianjin,
Zhejiang\}. These indices used to evaluate the
alternatives $z_i$ $(l = 1,2,3, ..., 16)$ are given below: $o_1$: labour
productivity of all staff (yuan per person); $o_2$: Profit each
$100$ of sales revenue (yuan); $o_3$: capital interest tax rate
(percent); $o_4$: profit rate of production (%), and $o_5$:
circulating investment with a value of $100$ yuan in factory
output. $o_5$ is a price-type indicator among them, whereas
others are profit-type indicator.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>$o_1$</th>
<th>$o_2$</th>
<th>$o_3$</th>
<th>$o_4$</th>
<th>$o_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_1$</td>
<td>26.446</td>
<td>2.38</td>
<td>10.16</td>
<td>9.85</td>
<td>26.80</td>
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<tr>
<td>$z_2$</td>
<td>47.177</td>
<td>8.89</td>
<td>16.61</td>
<td>15.77</td>
<td>31.05</td>
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<td>4.79</td>
<td>11.97</td>
<td>10.64</td>
<td>26.45</td>
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<td>$z_4$</td>
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<td>4.54</td>
<td>10.29</td>
<td>9.23</td>
<td>23.00</td>
</tr>
<tr>
<td>$z_5$</td>
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<td>3.17</td>
<td>8.13</td>
<td>9.17</td>
<td>29.25</td>
</tr>
<tr>
<td>$z_6$</td>
<td>26.925</td>
<td>3.06</td>
<td>9.34</td>
<td>10.84</td>
<td>30.11</td>
</tr>
<tr>
<td>$z_7$</td>
<td>30.721</td>
<td>4.15</td>
<td>10.87</td>
<td>11.44</td>
<td>30.36</td>
</tr>
<tr>
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<td>2.42</td>
<td>10.77</td>
<td>11.37</td>
<td>30.71</td>
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<tr>
<td>$z_9$</td>
<td>46.821</td>
<td>3.51</td>
<td>10.59</td>
<td>7.41</td>
<td>22.46</td>
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<td>2.58</td>
<td>8.25</td>
<td>8.62</td>
<td>32.57</td>
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<tr>
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<td>2.12</td>
<td>7.68</td>
<td>9.05</td>
<td>31.08</td>
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<td>$z_{12}$</td>
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<td>3.38</td>
<td>8.92</td>
<td>8.73</td>
<td>25.68</td>
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<tr>
<td>$z_{13}$</td>
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<td>6.06</td>
<td>13.84</td>
<td>12.87</td>
<td>26.55</td>
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<td>35.35</td>
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<td>29.80</td>
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<td>8.13</td>
<td>9.17</td>
<td>29.25</td>
</tr>
</tbody>
</table>

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<th>$o_4$</th>
<th>$o_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_1$</td>
<td>0.448</td>
<td>0.268</td>
<td>0.612</td>
<td>0.625</td>
<td>0.838</td>
</tr>
<tr>
<td>$z_2$</td>
<td>0.799</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.723</td>
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<tr>
<td>$z_3$</td>
<td>0.650</td>
<td>0.539</td>
<td>0.721</td>
<td>0.675</td>
<td>0.849</td>
</tr>
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<td>$z_4$</td>
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<td>0.511</td>
<td>0.620</td>
<td>0.585</td>
<td>0.977</td>
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<td>0.357</td>
<td>0.489</td>
<td>0.581</td>
<td>0.768</td>
</tr>
<tr>
<td>$z_6$</td>
<td>0.456</td>
<td>0.344</td>
<td>0.562</td>
<td>0.687</td>
<td>0.746</td>
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<tr>
<td>$z_7$</td>
<td>0.520</td>
<td>0.467</td>
<td>0.654</td>
<td>0.725</td>
<td>0.740</td>
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<tr>
<td>$z_8$</td>
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<td>0.272</td>
<td>0.648</td>
<td>0.721</td>
<td>0.731</td>
</tr>
<tr>
<td>$z_9$</td>
<td>0.793</td>
<td>0.395</td>
<td>0.638</td>
<td>0.470</td>
<td>1.000</td>
</tr>
<tr>
<td>$z_{10}$</td>
<td>0.394</td>
<td>0.290</td>
<td>0.497</td>
<td>0.547</td>
<td>0.690</td>
</tr>
<tr>
<td>$z_{11}$</td>
<td>0.489</td>
<td>0.238</td>
<td>0.462</td>
<td>0.574</td>
<td>0.723</td>
</tr>
<tr>
<td>$z_{12}$</td>
<td>0.658</td>
<td>0.380</td>
<td>0.537</td>
<td>0.554</td>
<td>0.875</td>
</tr>
<tr>
<td>$z_{13}$</td>
<td>1.000</td>
<td>0.682</td>
<td>0.833</td>
<td>0.816</td>
<td>0.846</td>
</tr>
<tr>
<td>$z_{14}$</td>
<td>0.366</td>
<td>0.524</td>
<td>0.430</td>
<td>0.715</td>
<td>0.635</td>
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<tr>
<td>$z_{15}$</td>
<td>0.734</td>
<td>0.411</td>
<td>0.547</td>
<td>0.535</td>
<td>0.754</td>
</tr>
<tr>
<td>$z_{16}$</td>
<td>0.479</td>
<td>0.357</td>
<td>0.489</td>
<td>0.581</td>
<td>0.768</td>
</tr>
</tbody>
</table>
\[
\mu = (0.36, 0.16, 0.16, 0.16, 0.16) \text{ and } \alpha = 0.2 \\
z_1(\mu) = OW_{A_{\mu}}(s_{11}, s_{12}, s_{13}, s_{14}, s_{15}) \\
= 0.36 \times 0.838 + 0.16 \times 0.625 + 0.16 \times 0.612 \\
+ 0.16 \times 0.448 + 0.16 \times 0.268 \\
= 0.6142
\]

Similarly,
\[
z_2(\mu) = 0.9235, \\
z_3(\mu) = 0.7192, \\
z_4(\mu) = 0.7833, \\
z_5(\mu) = 0.5814, \\
z_6(\mu) = 0.5964, \\
z_7(\mu) = 0.6450, \\
z_8(\mu) = 0.5931, \\
z_9(\mu) = 0.7274, \\
z_{10}(\mu) = 0.5249, \\
z_{11}(\mu) = 0.5424, \\
z_{12}(\mu) = 0.6556, \\
z_{13}(\mu) = 0.8683, \\
z_{14}(\mu) = 0.6278, \\
z_{15}(\mu) = 0.5814, \\
z_{16}(\mu) = 0.7043
\]

From the before calculation of \(z_l(\mu)\) for all \(l = 1, 2, \ldots, 16\). We arrange the values in descending order
\[
z_2 > z_{13} > z_4 > z_9 > z_3 > z_{16} > z_{12} > z_7 > z_{14} > z_1 > z_6 \\
> z_8 > z_5 > z_{15} > z_{11} > z_{10}
\]

RESULT

Based on above calculation results will be obtained that \(z_2\) has the highest value and good alternative decision making.

CONCLUSION

The process of economic benefits from provinces and municipalities in the central government test involves more parameters. In this paper we use OWA operator to solve MADM problem to determining its benefit. In this way we can analysis more decision-making issues like company, electrician, product, stock market, etc.

REFERENCES


