A Doubt Ꝁ − Ọ −Bipolar Fuzzy BCI-Ideals and Doubt Ꝁ − Ọ −Bipolar Fuzzy BCI-Implicative Ideals in BCI-Algebra

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Abstract

Abstract: This paper introduced the Doubt Ꝁ − Ọ-BFBCI-Ids and Doubt Ꝁ − Ọ-BFBCI-Imp-Ids with examples and properties are studied. In furthermore, discussed about Doubt Ꝁ − Ọ −Bipolar Fuzzy Union and Intersection set as its various algebraic aspects.

Keywords: Fuzzy Set (FS), Fuzzy BCI-ideal (FBCI-Id), Ꝁ − Ọ-Fuzzy Subset (Ꝁ − Ọ − FSb), Doubt Ꝁ − Ọ-Bipolar fuzzy set (Doubt Ꝁ − Ọ-BFS), Doubt Ꝁ − Q-bipolar fuzzy Ideal (DoubtꝀ − Ọ-BFI), Doubt Ꝁ − Ọ-bipolar fuzzy BCI-Ideal (DoubtꝀ − Ọ-BFBCI-Id) and Doubt Ꝁ − Ọ-bipolar fuzzy BCI-Implicative Ideal (Doubt Ꝁ − Ọ-BFBCI-Imp-Id).

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INTRODUCTION

[17]Zadeh L A described the notation of fuzzy sets in 1965. In 2004, Bipolar logic and bipolar fuzzy logic developed by [18]Yang Y. [19]Zimmermann H J initiated by the concept of Fuzzy set theory and its applications in 1985. In 1986, described the concept of Intuitionistic fuzzy sets by [1]Atanassov K T. [2]Hu Q P developed the concept of On BCI-algebras satisfying$(x * y) * z = x * (y * z)$ in 1980. [16]Nagarajan R, initiated by notation of a new structure and construction of Q-fuzzy groups in 2009. In 2019, Cubic intuitionistic structures applied to ideals of BCI-algebras developed by [15]Shum K P. [9]Aldhafeeri S depicted the concept of N-soft p-ideals of BCI-algebras in 2019. In 2009 introduced by the notation of BCI-Implicative ideals of BCI-algebras in [8]Meng J. [10]Jun Y B developed the concept of Hesitant fuzzy translations and extensions of subalgberas and ideals in BCK/BCI-algebras in 2017. Bipolar valued fuzzy sub algebras and bipolar fuzzy ideals of BCK/BCI-algebras in 2009 developed by [5]Lee K J. [7]Liu Y L described the notation of fuzzy ideals in BCI-algebras in 2001. Bipolar valued fuzzy sets and their operations developed by [6]Lee K M in 2000. [14]Premkumar M develop the concept of On Fundamental Algebraic Attributes of $\omega - Q$ −Fuzzy Subring, Normal Subring and Ideal in 2021. On κ − Q-Anti Fuzzy Normed Rings in 2021 described by [12]Prasanna A. [3]Iseki K initiated by the notation of BCI-algebras in 1980. Premkumar M's 2020[11] illustration of $\kappa - Q$ −Fuzzy Orders Relative to $\kappa - Q$ −Fuzzy Subgroups and Cyclic Group on a variety of essential characteristics. Fundamental Algebraic Properties on $\kappa - Q$ −Anti Fuzzy Normed Prime Ideal and $\kappa - Q$ −Anti Fuzzy Normed Maximal Ideal were developed by Premkumar M in 2021[13]. In 1993, Closed fuzzy ideals in BCI-algebras depicted by [4]Jun Y B.

In this paper introduced by the new contribution of Algebraic Properties on Doubt Ꝁ − Ọ-BFBCI-Ids. And also described the new notation of *Doubt* Ꝁ − Ǫ-BFBCI-Imp-Ids in BCI-Algebra and their results.

PRELIMINARIES

Definition: 3.1

An algebra $(G;*,0)$ of kind (2,0) is a BCI-algebra if it satisfies for all $x, y, z \in G$

(i) $((x*y)*(x*z))*(z*y)=0$

(ii) $(x*(x*y))*y=0$

(iii) $x*x=0$

(iv) $x*y=0 \text{ and } y*x=0 \Rightarrow x=y$.

Definition: 3.2

A FS μ in G is a FBCI-Id of G if it satisfies for all $x, y, z \in G$

(i) $\mu(0) \geq \mu(x)$

(ii) $\mu(x) \geq min\{\mu(x*y), \mu(y)\}$.

Definition: 3.3

A FS μ in G is a FBCI-Imp-Id of G if it satisfies for all $x, y, z \in G$

$$\mu\left\{\left(x*(y*(y*x))*\left(0*\left(0*(x*y)\right)\right)\right)\right\} \geq$$
$$min\left\{\mu\left(\left(((x*y,q)*y,q)*(0*y,q),q\right)*(z,q)\right), \mu(z)\right\}.$$

A Doubt Ꝁ − Ǫ-BFBCI-IDS AND Doubt Ꝁ − Ǫ-BFBCI-IMP-IDS IN BCI-ALGEBRA

Definition: 3.1

A Doubt Ꝁ − Ǫ-BFS, $\tilde{Ḁ}$ in G is called a Doubt Ꝁ − Ǫ-BFBCI-Id of G. If its following conditions

(a) (i) $\mu_{\tilde{Ḁ}^{Ꝁ-}}(0,ɋ) \leq \{(\mu_{\tilde{Ḁ}^-}(ũ,ɋ),Ꝁ)\}$

(ii) $\mu_{\tilde{Ḁ}^{Ꝁ+}}(0,ɋ) \geq \{(\mu_{\tilde{Ḁ}^+}(ũ,ɋ),Ꝁ)\}$

(b) (i) $\mu_{\tilde{Ḁ}^{Ꝁ-}}(ũ,ɋ) \leq max\{(\mu_{\tilde{Ḁ}^-}(ũ*ȅ,ɋ),Ꝁ),(\mu_{\tilde{Ḁ}^-}(ȅ,ɋ),Ꝁ)\}$

(ii) $\mu_{\tilde{Ḁ}^{Ꝁ+}}(ũ,ɋ) \geq min\{(\mu_{\tilde{Ḁ}^+}(ũ*ȅ,ɋ),Ꝁ),(\mu_{\tilde{Ḁ}^+}(ȅ,ɋ),Ꝁ)\}$, $\forall\, ũ, ȅ \in G$.

Definition: 3.2

A Doubt Ꝁ − Ǫ-BFS, $\tilde{Ḁ}$ in G is called a Doubt Ꝁ − Ǫ-BFBCI-Imp-Id of G if it satisfies in above definition condition (a) and the following conditions

(i) $\mu_{\tilde{Ḁ}^{Ꝁ-}}\{(ũ*(ȅ*(ȅ*ũ,ɋ),ɋ)*(0*(0*(ũ*ȅ,ɋ),ɋ),ɋ),ɋ)\} \leq max\left\{\left(\mu_{\tilde{Ḁ}^-}\left(\left(((ũ*ȅ,ɋ)*ȅ,ɋ)*(0*ȅ,ɋ),ɋ\right)*(z,ɋ)\right),Ꝁ\right),(\mu_{\tilde{Ḁ}^-}(z,ɋ),Ꝁ)\right\}$ and

(ii) $\mu_{\tilde{Ḁ}^{Ꝁ+}}\{(ũ*(ȅ*(ȅ*ũ,ɋ),ɋ)*(0*(0*(ũ*ȅ,ɋ),ɋ),ɋ),ɋ)\} \geq min\left\{\left(\mu_{\tilde{Ḁ}^+}\left(\left(((ũ*ȅ,ɋ)*ȅ,ɋ)*(0*ȅ,ɋ),ɋ\right)*(z,ɋ)\right),Ꝁ\right),(\mu_{\tilde{Ḁ}^+}(z,ɋ),Ꝁ)\right\}$, $\forall\, ũ, ȅ, z \in G$.

Example: 3.2.1.

Take into account the BCI-Algebra$(G,*,0)$, where $G=\{0,a,b,c\}$ and $*$ determined by the table.

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Let Doubt Ꝁ − Ǫ-BFS in G represented by

G	0	a	b	c
$\mu_{\tilde{Ḁ}^{Ꝁ-}}$	-0.8	-0.8	-0.5	-0.5
$\mu_{\tilde{Ḁ}^{Ꝁ+}}$	0.9	0.9	0.4	0.4

Then by routine calculations Doubt Ꝁ − Ǫ-BFBCI-Imp-Id of G.

Theorem: 3.3

Any Doubt Ꝁ − Ǫ-BFBCI-Imp-Id of G is a Ꝁ − Ǫ − BFI of G.

Proof:

Let, Doubt Ꝁ − Ǫ-BFBCI-Imp-Id of G

Then,

(i) $\mu_{\tilde{Ḁ}^{Ꝁ-}}\{(ũ*(ȅ*(ȅ*ũ,ɋ),ɋ)*(0*(0*(ũ*ȅ,ɋ),ɋ),ɋ),ɋ)\} \leq max\left\{\left(\mu_{\tilde{Ḁ}^-}\left(\left(((ũ*ȅ,ɋ)*ȅ,ɋ)*(0*ȅ,ɋ),ɋ\right)*(z,ɋ)\right),Ꝁ\right),(\mu_{\tilde{Ḁ}^-}(z,ɋ),Ꝁ)\right\}$ and

(ii) $\mu_{\tilde{Ḁ}^{Ꝁ+}}\{(ũ*(ȅ*(ȅ*ũ,ɋ),ɋ)*(0*(0*(ũ*ȅ,ɋ),ɋ),ɋ),ɋ)\} \geq min\left\{\left(\mu_{\tilde{Ḁ}^+}\left(\left(((ũ*ȅ,ɋ)*ȅ,ɋ)*(0*ȅ,ɋ),ɋ\right)*(z,ɋ)\right),Ꝁ\right),(\mu_{\tilde{Ḁ}^+}(z,ɋ),Ꝁ)\right\}$, $\forall\, ũ, ȅ, z \in G$.

Substitute z by ȅ and ȅ by 0 to get

(i) $\mu_{\tilde{Ḁ}^{Ꝁ-}}\{(ũ*(0*(0*ũ,ɋ),ɋ)*(0*(0*(ũ*0,ɋ),ɋ),ɋ),ɋ)\} \leq max\left\{\left(\mu_{\tilde{Ḁ}^-}\left(\left(((ũ*0,ɋ)*0,ɋ)*(0*0,ɋ),ɋ\right)*(ȅ,ɋ)\right),Ꝁ\right),(\mu_{\tilde{Ḁ}^-}(ȅ,ɋ),Ꝁ)\right\}$ and

(ii) $\mu_{\tilde{Ḁ}^{Ꝁ+}}\{(ũ*(0*(0*ũ,ɋ),ɋ)*(0*(0*(ũ*0,ɋ),ɋ),ɋ),ɋ)\} \geq min\left\{\left(\mu_{\tilde{Ḁ}^+}\left(\left(((ũ*0,ɋ)*0,ɋ)*(0*0,ɋ),ɋ\right)*(ȅ,ɋ)\right),Ꝁ\right),(\mu_{\tilde{Ḁ}^+}(ȅ,ɋ),Ꝁ)\right\}$, $\forall\, ũ, ȅ, z \in G$.

$\Rightarrow \mu_{\tilde{Ḁ}^{Ꝁ-}}(ũ,ɋ) \leq max\{(\mu_{\tilde{Ḁ}^-}(ũ*ȅ,ɋ),Ꝁ),(\mu_{\tilde{Ḁ}^-}(ȅ,ɋ),Ꝁ)\}$ and

$\mu_{\tilde{A}^{Ꝁ+}}(\tilde{u}, ꝗ) \geq min\{(\mu_{\tilde{A}^{+}}(\tilde{u} * \tilde{v}, ꝗ), Ꝁ), (\mu_{\tilde{A}^{+}}(\tilde{v}, ꝗ), Ꝁ)\}$, $\forall\, \tilde{u}, \tilde{v} \in G$.

Hence, Doubt Ꝁ − Ǫ-BFBCI-Id of G.

Example: 3.3.1

Take into account the BCI-Algebra$(G,*,0)$, where $G = \{0, a, b, c\}$ and $*$ determined by the table.

*	0	ď	ë	ғ
0	0	0	0	ғ
ď	ď	0	0	ғ
ë	ë	ë	0	ғ
ғ	ғ	ғ	ғ	0

Let Doubt Ꝁ − Ǫ-BFS in G represented by

G	**0**	ď	ë	ғ
$\mu_{\tilde{A}^{Ꝁ-}}$	-0.6	-0.4	-0.4	-0.4
$\mu_{\tilde{A}^{Ꝁ+}}$	0.8	0.7	0.7	0.7

Then not a Doubt Ꝁ − Ǫ-BFBCI-Imp-Id of G, as defined by $\mu_{\tilde{A}^{Ꝁ+}}\{(d * (e * (e * d, ꝗ), ꝗ) * (0 * (0 * (d * e, ꝗ), ꝗ), ꝗ), ꝗ)\} = \mu_{\tilde{A}^{Ꝁ+}}(d, ꝗ) = -0.4 \nleq -0.6 = min\left\{\left(\mu_{\tilde{A}^{+}}\left(\left(\left((d * e, ꝗ) * e, ꝗ\right) * (0 * e, ꝗ), ꝗ\right) * (0, ꝗ)\right), Ꝁ\right), (\mu_{\tilde{A}^{+}}(0, ꝗ), Ꝁ)\right\} = \mu_{\tilde{A}^{Ꝁ+}}(0, ꝗ)$.

Proposition: 3.4

Let, Doubt Ꝁ − Ǫ-BFS in G is a Doubt Ꝁ − Ǫ-BFBCI-Id of G, if and only if for all $\tilde{u}, \tilde{v}, z \in G$, $(\tilde{u} * \tilde{v}, ꝗ) * (z, ꝗ) = (0, ꝗ) \Rightarrow$

(i) $\mu_{\tilde{A}^{Ꝁ-}}(\tilde{u}, ꝗ) \leq max\{(\mu_{\tilde{A}^{-}}(\tilde{v}, ꝗ), Ꝁ), (\mu_{\tilde{A}^{-}}(z, ꝗ), Ꝁ)\}$ and

(ii) $\mu_{\tilde{A}^{Ꝁ+}}(\tilde{u}, ꝗ) \geq min\{(\mu_{\tilde{A}^{+}}(\tilde{v}, ꝗ), Ꝁ), (\mu_{\tilde{A}^{+}}(z, ꝗ), Ꝁ)\}$.

Proposition: 3.5

Let, Doubt Ꝁ − Ǫ-BFS in G is a Doubt Ꝁ − Ǫ-BFBCI-Id of G, if and only if for all $\tilde{u}, \tilde{v}, z \in G$, $(\tilde{u} * \tilde{v}, ꝗ) = 0 \Rightarrow$

(i) $\mu_{\tilde{A}^{Ꝁ-}}(\tilde{u}, ꝗ) \leq \mu_{\tilde{A}^{Ꝁ-}}(\tilde{v}, ꝗ)$ and
(ii) $\mu_{\tilde{A}^{Ꝁ+}}(\tilde{u}, ꝗ) \geq \mu_{\tilde{A}^{Ꝁ+}}(\tilde{v}, ꝗ)$.

Definition: 3.6

Let, two Doubt Ꝁ − Ǫ-BFSs in G. Then the union denoted by $\mu_{\tilde{A}_1}{}^{Ꝁ-} \cup \mu_{\tilde{A}_2}{}^{Ꝁ-}$ and $\mu_{\tilde{A}_1}{}^{Ꝁ+} \cup \mu_{\tilde{A}_2}{}^{Ꝁ+}$ is $min\left\{\mu_{\tilde{A}_1}{}^{Ꝁ-}, \mu_{\tilde{A}_2}{}^{Ꝁ-}\right\}$ and $max\left\{\mu_{\tilde{A}_1}{}^{Ꝁ+}, \mu_{\tilde{A}_2}{}^{Ꝁ+}\right\}$.

Definition: 3.7

Let, two Doubt Ꝁ − Ǫ-BFSs in G. Then the intersection denoted by $\mu_{\tilde{A}_1}{}^{Ꝁ-} \cap \mu_{\tilde{A}_2}{}^{Ꝁ-}$ and $\mu_{\tilde{A}_1}{}^{Ꝁ+} \cap \mu_{\tilde{A}_2}{}^{Ꝁ+}$ is $max\left\{\mu_{\tilde{A}_1}{}^{Ꝁ-}, \mu_{\tilde{A}_2}{}^{Ꝁ-}\right\}$ and $min\left\{\mu_{\tilde{A}_1}{}^{Ꝁ+}, \mu_{\tilde{A}_2}{}^{Ꝁ+}\right\}$.

Theorem: 3.8

Let, two Doubt Ꝁ − Ǫ-BFSs in G and two Doubt Ꝁ − Ǫ-BFBCI-Imp-Id of G. Then $\mu_{\tilde{A}_1}{}^{Ꝁ-} \cup \mu_{\tilde{A}_2}{}^{Ꝁ-}$ and $\mu_{\tilde{A}_1}{}^{Ꝁ+} \cup \mu_{\tilde{A}_2}{}^{Ꝁ+}$ is a Doubt Ꝁ − Ǫ-BFBCI-Imp-Ids of G.

Proof:

Let two Doubt Ꝁ − Ǫ-BFBCI-Imp-Ids of G.

Then,

(i) $\mu_{\tilde{A}_1}{}^{Ꝁ-}(0, ꝗ) \leq \mu_{\tilde{A}_1}{}^{-}((\tilde{u}, ꝗ), Ꝁ)$ and $\mu_{\tilde{A}_2}{}^{Ꝁ-}(0, ꝗ) \leq \mu_{\tilde{A}_2}{}^{-}((\tilde{u}, ꝗ), Ꝁ)$

(ii) $\mu_{\tilde{A}_1}{}^{Ꝁ+}(0, ꝗ) \geq \mu_{\tilde{A}_1}{}^{+}((\tilde{u}, ꝗ), Ꝁ)$ and $\mu_{\tilde{A}_2}{}^{Ꝁ+}(0, ꝗ) \geq \mu_{\tilde{A}_2}{}^{+}((\tilde{u}, ꝗ), Ꝁ)$.

Therefore

$min\left\{\mu_{\tilde{A}_1}{}^{Ꝁ-}, \mu_{\tilde{A}_2}{}^{Ꝁ-}\right\}(0, ꝗ) \leq min\{\mu_{\tilde{A}_1}{}^{-}((\tilde{u}, ꝗ), Ꝁ), \mu_{\tilde{A}_2}{}^{-}((\tilde{u}, ꝗ), Ꝁ)\}$ and

$max\left\{\mu_{\tilde{A}_1}{}^{Ꝁ+}, \mu_{\tilde{A}_2}{}^{Ꝁ+}\right\}(0, ꝗ) \geq max\left\{\mu_{\tilde{A}_1}{}^{+}((\tilde{u}, ꝗ), Ꝁ), \mu_{\tilde{A}_2}{}^{+}((\tilde{u}, ꝗ), Ꝁ)\right\}$

For all $\tilde{u}, \tilde{v} \in G$ *and* $ꝗ \in Ǫ$,

$\mu_{\tilde{A}_1}{}^{Ꝁ+}\{(\tilde{u} * (\tilde{v} * (\tilde{v} * \tilde{u}, ꝗ), ꝗ) * (0 * (0 * (\tilde{u} * \tilde{v}, ꝗ), ꝗ), ꝗ), ꝗ)\} = \mu_{\tilde{A}_1}{}^{+}\left\{\left(\left((\tilde{u} * \tilde{v}, ꝗ) * \tilde{v}, ꝗ\right) * (0 * \tilde{v}, ꝗ), ꝗ\right), Ꝁ\right\}$,

$\mu_{\tilde{A}_1}{}^{Ꝁ-}\{(\tilde{u} * (\tilde{v} * (\tilde{v} * \tilde{u}, ꝗ), ꝗ) * (0 * (0 * (\tilde{u} * \tilde{v}, ꝗ), ꝗ), ꝗ), ꝗ)\} = \mu_{\tilde{A}_1}{}^{-}\left\{\left(\left((\tilde{u} * \tilde{v}, ꝗ) * \tilde{v}, ꝗ\right) * (0 * \tilde{v}, ꝗ), ꝗ\right), Ꝁ\right\}$ and

$\mu_{\tilde{A}_2}{}^{Ꝁ+}\{(\tilde{u} * (\tilde{v} * (\tilde{v} * \tilde{u}, ꝗ), ꝗ) * (0 * (0 * (\tilde{u} * \tilde{v}, ꝗ), ꝗ), ꝗ), ꝗ)\} = \mu_{\tilde{A}_2}{}^{+}\left\{\left(\left((\tilde{u} * \tilde{v}, ꝗ) * \tilde{v}, ꝗ\right) * (0 * \tilde{v}, ꝗ), ꝗ\right), Ꝁ\right\}$,

$\mu_{\tilde{A}_2}{}^{Ꝁ-}\{(\tilde{u} * (\tilde{v} * (\tilde{v} * \tilde{u}, ꝗ), ꝗ) * (0 * (0 * (\tilde{u} * \tilde{v}, ꝗ), ꝗ), ꝗ), ꝗ)\} = \mu_{\tilde{A}_2}{}^{-}\left\{\left(\left((\tilde{u} * \tilde{v}, ꝗ) * \tilde{v}, ꝗ\right) * (0 * \tilde{v}, ꝗ), ꝗ\right), Ꝁ\right\}$

Thus, $max\left\{\mu_{\tilde{A}_1}{}^{Ꝁ+}, \mu_{\tilde{A}_2}{}^{Ꝁ+}\right\}\{(\tilde{u} * (\tilde{v} * (\tilde{v} * \tilde{u}, ꝗ), ꝗ) * (0 * (0 * (\tilde{u} * \tilde{v}, ꝗ), ꝗ), ꝗ), ꝗ)\} = max\left\{\mu_{\tilde{A}_1}{}^{+}\left(\left(\left((\tilde{u} * \tilde{v}, ꝗ) * \tilde{v}, ꝗ\right) * (0 * \tilde{v}, ꝗ), ꝗ\right), Ꝁ\right), \mu_{\tilde{A}_2}{}^{+}\left(\left(\left((\tilde{u} * \tilde{v}, ꝗ) * \tilde{v}, ꝗ\right) * (0 * \tilde{v}, ꝗ), ꝗ\right), Ꝁ\right)\right\} = max\left\{\mu_{\tilde{A}_1}{}^{+}, \mu_{\tilde{A}_2}{}^{+}\right\}\left\{\left(\left((\tilde{u} * \tilde{v}, ꝗ) * \tilde{v}, ꝗ\right) * (0 * \tilde{v}, ꝗ), ꝗ\right), Ꝁ\right\}$,

and

$$\min\left\{\mu_{\tilde{A}_1}{}^{Ꝁ-},\mu_{\tilde{A}_2}{}^{Ꝁ-}\right\}\{(\tilde{u}*(\ddot{v}*(\ddot{v}*\tilde{u},q),q)*(0*(0*(\tilde{u}*\ddot{v},q),q),q),q)\}$$
$$=\min\left\{\mu_{\tilde{A}_1}{}^{-}\left(\left(\left(\left((\tilde{u}*\ddot{v},q)*\ddot{v},q\right)*(0*\ddot{v},q),q\right),Ꝁ\right),\mu_{\tilde{A}_2}{}^{-}\left(\left(\left(\left((\tilde{u}*\ddot{v},q)*\ddot{v},q\right)*(0*\ddot{v},q),q\right),Ꝁ\right)\right)\right\}$$
$$=\min\{\mu_{\tilde{A}_1}{}^{-},\mu_{\tilde{A}_2}{}^{-}\}\left\{\left(\left(\left((\tilde{u}*\ddot{v},q)*\ddot{v},q\right)*(0*\ddot{v},q),q\right),Ꝁ\right\}.$$

That is $\mu_{\tilde{A}_1}{}^{Ꝁ-}\cup\mu_{\tilde{A}_2}{}^{Ꝁ-}$ and $\mu_{\tilde{A}_1}{}^{Ꝁ+}\cup\mu_{\tilde{A}_2}{}^{Ꝁ+}$ is Ꝁ − Ǫ-BFBCI-Imp-Ids of G.

Theorem: 3.9

Let, two Doubt Ꝁ − Ǫ-BFSs in G, and two Doubt Ꝁ − Ǫ-BFBCI-Imp-Ids of G. Then $\mu_{\tilde{A}_1}{}^{Ꝁ-}\cap\mu_{\tilde{A}_2}{}^{Ꝁ-}$ and $\mu_{\tilde{A}_1}{}^{Ꝁ+}\cap\mu_{\tilde{A}_2}{}^{Ꝁ+}$ is a Doubt Ꝁ − Ǫ-BFBCI-Imp-Ids of G.

Proof:

Let, two Doubt Ꝁ − Ǫ-BFBCI-Imp-Ids of G

Then,

(i) $\mu_{\tilde{A}_1}{}^{Ꝁ-}(0,q)\le\mu_{\tilde{A}_1}{}^{-}\left((\tilde{u},q),Ꝁ\right)$ and $\mu_{\tilde{A}_2}{}^{Ꝁ-}(0,q)\le\mu_{\tilde{A}_2}{}^{-}\left((\tilde{u},q),Ꝁ\right)$

(ii) $\mu_{\tilde{A}_1}{}^{Ꝁ+}(0,q)\ge\mu_{\tilde{A}_1}{}^{+}\left((\tilde{u},q),Ꝁ\right)$ and $\mu_{\tilde{A}_2}{}^{Ꝁ+}(0,q)\ge\mu_{\tilde{A}_2}{}^{+}\left((\tilde{u},q),Ꝁ\right)$.

Therefore

$$\max\left\{\mu_{\tilde{A}_1}{}^{Ꝁ-},\mu_{\tilde{A}_2}{}^{Ꝁ-}\right\}(0,q)\le\max\{\mu_{\tilde{A}_1}{}^{-}((\tilde{u},q),Ꝁ),\mu_{\tilde{A}_2}{}^{-}((\tilde{u},q),Ꝁ)\}$$ and
$$\min\left\{\mu_{\tilde{A}_1}{}^{Ꝁ+},\mu_{\tilde{A}_2}{}^{Ꝁ+}\right\}(0,q)\ge\min\left\{\mu_{\tilde{A}_1}{}^{+}((\tilde{u},q),Ꝁ),\mu_{\tilde{A}_2}{}^{+}((\tilde{u},q),Ꝁ)\right\}$$

For all $\tilde{u},\ddot{v}\in G$ *and* $q\in Ǫ$,

$$\mu_{\tilde{A}_1}{}^{Ꝁ+}\{(\tilde{u}*(\ddot{v}*(\ddot{v}*\tilde{u},q),q)*(0*(0*(\tilde{u}*\ddot{v},q),q),q),q)\}=\mu_{\tilde{A}_1}{}^{+}\left\{\left(\left(\left((\tilde{u}*\ddot{v},q)*\ddot{v},q\right)*(0*\ddot{v},q),q\right),Ꝁ\right\},$$
$$\mu_{\tilde{A}_1}{}^{Ꝁ-}\{(\tilde{u}*(\ddot{v}*(\ddot{v}*\tilde{u},q),q)*(0*(0*(\tilde{u}*\ddot{v},q),q),q),q)\}=\mu_{\tilde{A}_1}{}^{-}\left\{\left(\left(\left((\tilde{u}*\ddot{v},q)*\ddot{v},q\right)*(0*\ddot{v},q),q\right),Ꝁ\right\}$$ and
$$\mu_{\tilde{A}_2}{}^{Ꝁ+}\{(\tilde{u}*(\ddot{v}*(\ddot{v}*\tilde{u},q),q)*(0*(0*(\tilde{u}*\ddot{v},q),q),q),q)\}=\mu_{\tilde{A}_2}{}^{+}\left\{\left(\left(\left((\tilde{u}*\ddot{v},q)*\ddot{v},q\right)*(0*\ddot{v},q),q\right),Ꝁ\right\},$$
$$\mu_{\tilde{A}_2}{}^{Ꝁ-}\{(\tilde{u}*(\ddot{v}*(\ddot{v}*\tilde{u},q),q)*(0*(0*(\tilde{u}*\ddot{v},q),q),q),q)\}=\mu_{\tilde{A}_2}{}^{-}\left\{\left(\left(\left((\tilde{u}*\ddot{v},q)*\ddot{v},q\right)*(0*\ddot{v},q),q\right),Ꝁ\right\}$$

Thus, $\min\left\{\mu_{\tilde{A}_1}{}^{Ꝁ+},\mu_{\tilde{A}_2}{}^{Ꝁ+}\right\}\{(\tilde{u}*(\ddot{v}*(\ddot{v}*\tilde{u},q),q)*(0*(0*(\tilde{u}*\ddot{v},q),q),q),q)\}=\min\left\{\mu_{\tilde{A}_1}{}^{+}\left(\left(\left(\left((\tilde{u}*\ddot{v},q)*\ddot{v},q\right)*(0*\ddot{v},q),q\right),Ꝁ\right),\mu_{\tilde{A}_2}{}^{+}\left(\left(\left(\left((\tilde{u}*\ddot{v},q)*\ddot{v},q\right)*(0*\ddot{v},q),q\right),Ꝁ\right)\right\}=\min\left\{\mu_{\tilde{A}_1}{}^{+},\mu_{\tilde{A}_2}{}^{+}\right\}\left\{\left(\left(\left((\tilde{u}*\ddot{v},q)*\ddot{v},q\right)*(0*\ddot{v},q),q\right),Ꝁ\right\},$

and

$$\max\left\{\mu_{\tilde{A}_1}{}^{Ꝁ-},\mu_{\tilde{A}_2}{}^{Ꝁ-}\right\}\{(\tilde{u}*(\ddot{v}*(\ddot{v}*\tilde{u},q),q)*(0*(0*(\tilde{u}*\ddot{v},q),q),q),q)\}$$
$$=\max\left\{\mu_{\tilde{A}_1}{}^{-}\left(\left(\left(\left((\tilde{u}*\ddot{v},q)*\ddot{v},q\right)*(0*\ddot{v},q),q\right),Ꝁ\right),\mu_{\tilde{A}_2}{}^{-}\left(\left(\left(\left((\tilde{u}*\ddot{v},q)*\ddot{v},q\right)*(0*\ddot{v},q),q\right),Ꝁ\right)\right)\right\}$$
$$=\max\{\mu_{\tilde{A}_1}{}^{-},\mu_{\tilde{A}_2}{}^{-}\}\left\{\left(\left(\left((\tilde{u}*\ddot{v},q)*\ddot{v},q\right)*(0*\ddot{v},q),q\right),Ꝁ\right\}.$$

That is $\mu_{\tilde{A}_1}{}^{Ꝁ-}\cap\mu_{\tilde{A}_2}{}^{Ꝁ-}$ and $\mu_{\tilde{A}_1}{}^{Ꝁ+}\cap\mu_{\tilde{A}_2}{}^{Ꝁ+}$ is Doubt Ꝁ − Ǫ-BFBCI-Imp-Ids of G.

Conclusions

During in this paper, we acquainted a Doubt Ꝁ − Q-BFBCI-Id of Fuzzy BCI -algebra which is discussed with illustrative examples and proposition of Algebras and also investigated Doubt Ꝁ − Q-BFBCI-Imp-Ids. In further future work define as Doubt Doubt Ꝁ − Q-BFBCI-Id and Doubt Ꝁ − Q-BFBCI-Imp-Ids.

Hat tip

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