Original Article

OPTIMIZATION TECHNIQUES IN TRANSPORTATION PROBLEM

Shalini. K[1], Noor Jahan[2], Sridevi. Polasi[3]

[1] Department of Mathematics, REVA University, Rukmini Knowledge Park, Kattigenahalli, Yelahanka, Bangalore, Karnataka-560064, India.
shalinimadhus13@gmail.com
[2] Assistant Professor, Department of Mathematics, HKBK Degree College, Bangalore.
drnoorjahan2020@gmail.com
[3] Associate Professor, Department of Mathematics, REVA University, Rukmini Knowledge Park, Kattigenahalli, Yelahanka, Bangalore, Karnataka-560064, India.
p.sridevi@reva.edu.in

DOI: 10.47750/pnr.2022.13.S07.354

Abstract

In this paper, the best optimality condition has been checked. Thus, optimizing the transportation problem of variables has remarkably been significant to various disciplines. Here we implemented different types of transportation problems like the northwest corner method, least cost method, Vogel's Approximation Method (VAM), and MODI method. Then we model the Transportation Problem to LPP, then solution by different suitable methods like Simplex and Goal Programming model and we analyze those answers.

Keywords: Transportation model, Simplex method, Goal programming,

Introduction

The transportation model is a special type of networks problems that for shipping a commodity from source to destination. The transportation model deals with getting the least cost plan to transport a commodity from a number of sources (m) to the number of destinations (n). Let x_i be the number of supply units required at source i (i=1, 2, 3....... m), dj is the quantity of requested units required at the endpoint j (j=1, 2,3.... n) and C_{ij} denote the unit transportation cost for transporting the units from starting point i to endpoint j. Using the linear programming method to solve the transportation problem, we determine the value of the objective function which minimizes the cost of transporting, and also determines the number of units that can be transported from starting point i to endpoint j. If x_{ij} is the number of units transported from starting point i to endpoint j. x_{ij} is greater than equal to 0 for all starting point (i) to end point (j). A transportation problem is said to be balanced if the supply from all sources equals the total demand in all destinations. If $x_{ij} < 0$ for all i to j it is called unbalanced.

The simplex method is one of the approaches to solving linear programming problems. In 1947 simplex method was invented by George Dantzig. A three dimensions simplex is a four-sided pyramid having four corners. Simplex finds the best corner of the feasible region. It usually starts with the corner point which has no values then it moves to the neighbourhood point which improves the solution simplex repeats the process until gets the optimum solution. Goal Programming is an extension of linear programming problems to manage multiple conflicting objective events. Each of these events is given a goal or target value to be attained. Unwanted deviations from this set of goal values are then minimized in an achievement function. The transportation problem was first developed in 1781 by the French mathematician Gaspard Monge. In World War II Major advances were made by the Soviet mathematician and economist Leonid Kantorovich. The transportation problem is sometimes known as the Monge-Kantorovich transportation problem.

In 1954 Charnes and Cooper [4] developed the goal programming problem. Dantzing has used a simplex method to the transportation problems as the primal simplex transportation method. Lee and Ignizio studied a goal programming approach to solve transportation problems. Agarana et. all [1] implemented transportation problems for the movement of people and goods in a Potential World Class University. Arthur et. all [2] developed a flow model for a chemical and pharmaceutical company. Ashton et. all [3] used a Goal programming model for financial problems. Frank et. all [6] implemented the distribution of a product from several sources to numerous localities. Kirca and Statir [7] developed an initial solution for the transportation problem. Kwak et. all [8] used a goal programming model for the improved transportation problem. Sridevi. Polasi et. all [9] used the goal programming method for the allocation of working hours in plant tissue culture. Vishwas Deep Joshi and Nilama Gupta [10] discussed the identification of a more-for-less paradox in the linear fractional transportation problem using an objective matrix.

Goals:

- Solving Transportation Problem by Multi Criteria Decision Making (MCDM) methods and analyze the optimal solution.
- Model the Transportation Problem to LPP finding solution by different suitable methods like Graphical, Simplex etc. and analyze those answers.
- Identify the best MCDM method to solve Transportation Problems.

Problem Formulation:

Karnataka state ships truckloads of coal from three different coal companies to four thermal power plants. The supply (in truckloads) and the demand (also in truckloads) together with the unit transportation costs (in thousands) per truckload on the different routes are summarized in the transportation model in table.

Companies/Power plants	Raichur Thermal power Plant	Yermarus Thermal power Plant	Udupi Thermal power Plant	Bellary Thermal power Plant	Available
Singareni Collieries Company Limited	11	13	17	14	250
Mahanadi Coalfield Limited	16	18	14	10	300
Western Coalfield Limited	21	24	13	10	400
Requirement	200	225	275	250	950

Methods to Solve:

There are three methods to determine the solution for balanced transportation problem:

1. North West Corner Method.

2. Least Cost Method.

3. Vogel’s Approximation Method (VAM).

The three methods differ in the "quality" of the starting basic solution they produce and better starting solution yields a smaller objective value. We present the three methods and an illustrative example is solved by these three methods.

1. North-west Corner Method:

Definition: The North West corner rule is a technique for calculating an initial feasible solution for a transportation problem. The method starts at the North west-corner method (route) of the tableau (variable x_{11}).

i. The first allocation is made in the cell (1,1), the magnitude being

x_{11}= min(250,200) = 200.

ii. The Second allocation is made in the cell (1,2) and the magnitude of the allocation is given by x_{12} = min(250-200,225) =50.

iii. Third allocation is made in the cell (2,2) and the magnitude of the allocation is given by x_{22} = min (300,225-50) =175.

iv. Fourth allocation is made in the cell (2,3) and the magnitude of the allocation is given by x_{23} = min (300-175,275) =125.

v. Fifth allocation is made in the cell (3,3) and the magnitude of the allocation is given by x_{33} = min (400,275-125) =150.

vi. Sixth allocation is made in the cell (3,4) and the magnitude of the allocation is given by x_{34} = min (400-150,250) =250.

Companies/Power plants	Raichur Thermal power Plant	Yermarus Thermal power Plant	Udupi Thermal power Plant	Bellary Thermal power Plant	Available
Singareni Collieries Company Limited	11(200)	13(50)	17	14	250-200=50
Mahanadi Coalfield Limited	16	18(175)	14(125)	10	300-175=125
Western Coalfield Limited	21	24	13(150)	10(250)	400-150=250
Requirement	200	225-50=175	275-125=150	250	950

Total Cost Z = 200×11 + 50×13 + 175×18 + 125×14 + 150×13 +250×10 = 12, 200

The Transportation Cost using North West Corner Method is Rs.12, 20, 00,000.

2. Least-Cost Method:

Definition: The Least Cost Method is another method used to obtain the initial feasible solution for the transportation problem

Companies/Power plants	Raichur Thermal power Plant	Yermarus Thermal power Plant	Udupi Thermal power Plant	Bellary Thermal power Plant	Available
Singareni Collieries Company Limited	11	13	17	14	250
Mahanadi Coalfield Limited	16	18	14	10	300
Western Coalfield Limited	21	24	13	10	400
Requirement	200	225	275	250	950

1. In the above table Cell (3,4) has the least unit cost in the tableau =10.

2. The most that can be shipped through (3,4) is x_{12}= min (250,300) = 250.

3. Which happens to satisfy column 4 simultaneously, we arbitrarily cross out column 4and adjust in the availability 400-250= 50.

4. Cell(1,1) has the least unit cost in the tableau 11.

5. The most that can be shipped through (1,1) is x_{11}= min (200,250) = 200. which happens to satisfy column 1 simultaneously,

6. We arbitrarily cross out column 1 and adjust the in availability 250- 200=50.

7. Continuing in the same manner, we successively assign Cell (1,2) has the least unit cost in the tableau 13.

8. The most that can be shipped through (1,2) is x_{12}=min (225,50) =50.

9. Which happens to satisfy row 1 simultaneously, we arbitrarily cross out row 1 and adjust the in requirement 225-50= 175

10. Continuing in the same manner, we successively assign Cell (3,3) has the least unit cost in the tableau 13.

11. The most that can be shipped through (3,3) is x_{33}=min (275,400) =275.

12. Which happens to satisfy column 3 simultaneously, we arbitrarily cross out column 3 and adjust the in-availability 400-275=125

13. Continuing in the same manner, we successively assign Cell (2,2) has the least unit cost in the tableau 18.

14. The most that can be shipped through (2,2) is x_{22}=min (50,175) =50.

15. Which happens to satisfy row 2 simultaneously, we arbitrarily cross out row 2 and adjust the in requirement 175-50=125

16. Continuing in the same manner, we successively assign Cell (3,2) has the least unit cost in the tableau 24.

17. The most that can be shipped through (3,2) is x_{32}=min (125,125) =125.

18. Which happens to satisfy row 3 simultaneously, we arbitrarily cross out row 3 and balance the availability and requirement.

Companies/Power plants	Raichur Thermal power Plant	Yermarus Thermal power Plant	Udupi Thermal power Plant	Bellary Thermal power Plant	Available
Singareni Collieries Company Limited	11(200)	13(50)	17	14	250-50=200
Mahanadi Coalfield Limited	16	18(175)	14(125)	10	300-125=175
Western Coalfield Limited	21	24	13(150)	10(250)	400-250=150
Requirement	200	225-175=50	275-150=125	250-250=0	950

Total Cost Z=200×11 + 50×13 +175×18 + 125×14 + 150×13 +250×10 = 12, 200.

The Transportation Cost using Least Cost Method is Rs. 12, 20, 00,000.

3. Vogel's Approximation Method (VAM):

Vogel's Approximation Method is an improved version of the minimum-cost method that generally produces better starting solutions.

Companies/Power plants	Raichur Thermal power Plant	Yermarus Thermal power Plant	Udupi Thermal power Plant	Bellary Thermal power Plant	Available
Singareni Collieries Company Limited	11	13	17	14	250
Mahanadi Coalfield Limited	16	18	14	10	300
Western Coalfield Limited	21	24	13	10	400
Requirement	200	225	275	250	950

- ❖ In row A, 11 is the least value and 13 is the second least value and their absolute difference is 2.
- ❖ Similarly, for row B,10 is the value and14 is the second least value and their absolute difference is 4.
- ❖ For row C,10 is the value and13 is the second least value and their absolute difference is 3.
- ❖ For each column find the least value and then the second least value and take the absolute difference of these two least values then write it in the corresponding column difference.
- ❖ In column D, 11 is the least value and 16 is the second least value and their absolute difference is 5.
- ❖ Similarly, in column E,13 is the least value and 18 is the second least value and their absolute difference is 5.

- ❖ In column F,13 is the least value and 14 is the second least value and their absolute difference is 1.
- ❖ In column G,10 is the least value and 10 is the second least value and their absolute difference is 0.

Companies/Power plants	Raichur Thermal power Plant	Yermarus Thermal power Plant	Udupi Thermal power Plant	Bellary Thermal power Plant	Available	P_1
Singareni Colliers Company Limited	11(200)	13	17	14	250-200=50	2
Mahanadi Coalfield Limited	16	18	14	10	300	4
Western Coalfield Limited	21	24	13	10	400	3
Requirement	200	225	275	250	950	
P_1	5	5	1	0		

These values of row difference and column difference are also called as penalty.

❑ Now find the cell with the least cost in row B and allocate the minimum among the available of the respective row and the requirement of the respective column.

❑ Requirement is smaller than the available so allocate the column's i.e., requirement 200 to the cell. Then cancel the column D.

Similarly, from the remaining cells, find out the row difference and column difference.

- ❖ In row A, 13 is the least value and 14 is the second least value and their absolute difference is 1.
- ❖ Similarly, for row B, 10 is the value and 14 is the second least value and their absolute difference is 4.
- ❖ For row C,10 is the value and 13 is the second least value and their absolute difference is 3.
- ❖ For each column find the least value and then the second least value and take the absolute difference of these two least values then write it in the corresponding column difference.
- ❖ In column E, 13 is the least value and 18 is the second least value and their absolute difference is 5.
- ❖ Similarly, in column F, 13 is the least value and 14 is the second least value and their absolute difference is 1.
- ❖ In column G, 10 is the least value and 10 is the second least value and their absolute difference is 0.

Companies/Power plants	Raichur Thermal power Plant	Yermarus Thermal power Plant	Udupi Thermal power Plant	Bellary Thermal power Plant	Available	P_1	P_2
Singareni Collieries	11(200)	13(50)	17	14	250-	2	1

Company Limited					200=50		
Mahanadi Coalfield Limited	16	18	14	10	300	4	4
Western Coalfield Limited	21	24	13	10	400	3	3
Requirement	200	225-50=175	275	250	950		
P_1	5	5	1	0			
P_2		5	1	0			

Similarly, again from the remaining cells, find out the row difference and column difference.

- ❖ In row B, 10 is the least value and 14 is the second least value and their absolute difference is 4.
- ❖ Similarly, for row C, 10 is the value and 13 is the second least value and their absolute difference is 3.
- ❖ For each column find the least value and then the second least value and take the absolute difference of these two least values then write it in the corresponding column difference.
- ❖ In column E, 18 is the least value and 24 is the second least value and their absolute difference is 6.
- ❖ Similarly, in column F, 13 is the least value and 14 is the second least value and their absolute difference is 1.
- ❖ In column G, 10 is the least value and 10 is the second least value and their absolute difference is 0.

Companies/Power plants	Raichur Thermal power Plant	Yermarus Thermal power Plant	Udupi Thermal power Plant	Bellary Thermal power Plant	Available	P 1	P_2	P_3
Singareni Collieries Company Limited	11(200)	13(50)	17	14	250-200=50	2	1	
Mahanadi Coalfield Limited	16	18(175)	14	10	300	4	4	4
Western Coalfield Limited	21	24	13	10	400	3	3	3
Requirement	200	225-50=175	275	250	950			
P_1	5	5	1	0				
P_2		5	1	0				
P_3		6	1	0				

Similarly, again from the remaining cells, find out the row difference and column difference.

- In row B, 10 is the least value and 14 is the second least value and their absolute difference is 4.
- Similarly, for row C, 10 is the value and 13 is the second least value and their absolute difference is 3.
- For each column find the least value and then the second least value and take the absolute difference of these two least values then write it in the corresponding column difference.
- Similarly, in column F, 13 is the least value and 14 is the second least value and their absolute difference is 1.
- In column G, 10 is the least value and 10 is the second least value and their absolute difference is 0.

Companies/Power plants	Raichur Thermal power Plant	Yermarus Thermal power Plant	Udupi Thermal power Plant	Bellary Thermal power Plant	Available	P_1	P_2	P_3	P_4
Singareni Collieries Company Limited	11(200)	13(50)	17	14	250-200=50	2	1		
Mahanadi Coalfield Limited	16	18(175)	14	10(125)	300-175=125	4	4	4	4
Western Coalfield Limited	21	24	13	10	400	3	3	3	3
Requirement	200	225-50=175	275	250	950				
P_1	5	5	1	0					
P_2		5	1	0					
P_3		6	1	0					
P_4			1	0					

The transportation cost according to this route is given by

Z=200×11 + 50×13 + 175×18 + 125×10 + 275×13 +12 5×10 =12075.

The Transportation Cost using Vogel's Approximation Method is Rs. 12, 07,50,000.

MODI Method – UV Method:

Step 1: Check whether the problem is balanced or not.

Companies/Power plants	Raichur Thermal power Plant	Yermarus Thermal power Plant	Udupi Thermal power Plant	Bellary Thermal power Plant	Available

Singareni Collieries Company Limited	11	13	17	14	250
Mahanadi Coalfield Limited	16	18	14	10	300
Western Coalfield Limited	21	24	13	10	400
Requirement	200	225	275	250	950

Step 2: Finding the initial basic feasible solution.

Companies/Power plants	Raichur Thermal power Plant	Yermarus Thermal power Plant	Udupi Thermal power Plant	Bellary Thermal power Plant	Available
Singareni Collieries Company Limited	11(200)	13(50)	17	14	250-200=50
Mahanadi Coalfield Limited	16	18(175)	14(125)	10	300-175=125
Western Coalfield Limited	21	24	13(150)	10(250)	400-150=250
Requirement	200	225-50=175	275-125=150	250	950

The Transportation cost according to the above route is given by

Z=200×11 + 50×13 + 175×18 + 125×14 + 150×13 +250×10 =12,200.

The Transportation cost using MODI method = Rs. 12,20,00,000

Step 3: U-V method to optimize the initial basic feasible solution.

The following is the initial basic feasible solution:

11(200)	13(50)	17	14
16	18(175)	14(125)	10
21	24	13(150)	10(250)

Let $u_1 = 0$. Then using the formula $u_i + v_j = C_{ij}$. We will get

$u_1 + v_1 = 11$, $0 + v_1 = 11$, $v_1 = 11$ (i.e., C_{11})

$u_1 + v_2=13$, $0+ v_2=13$, $v_2=13$ (i.e., C_{12})

$u_2 + v_3=14$, $5+ v_3=14$, $v_3= 9$ (i.e., C_{23})

$u_3 + v_3=13$, $u_3+9=13$, $u_3=4$(i.e., C_{33})

$u_3 + v_4=10$, $4+ v_4=10$, $v_4= 6$ (i.e., C_{34})

	$v_1=11$	$v_2=13$	$v_3=9$	$v_4=6$
$U_1=0$	11(200)	13(50)	17	14
$U_2=5$	16	18(175)	14(125)	10
$U_3=4$	21	24	13(150)	10(250)

Now, compute the non-occupied cell using the formula $P_{ij} = u_i+ v_j– C_{ij}$. Let's compute this one by one.

1. For C_{13}, $P_{13} = 0 + 9 -17= -8$ (here $C_{13} = 17$, $u_1 = 0$ and $v_3 = 9$)
2. For C_{14}, $P_{14} = 0 + 6 - 14 = - 8$ (here $C_{14} = 14$, $u_1 = 0$ and $v_4 = 6$)
3. For C_{21}, $P_{21} = 5 + 11 – 16 = 0$ (here $C_{21} = 16$, $u_2= 5$ and $v_1 = 11$)
4. For C_{24}, $P_{24}= 5 + 6 – 10 = 1$ (here $C_{24} = 10$, $u_2 = 5$ and $v_4 = 6$)
5. For C_{31}, $P_{31} = 4 + 11 – 21 = - 6$(here $C_{31} = 21$, $u_3 = 4$ and $v_1 = 11$)
6. For C_{32}, $P_{32} = 4 + 13 – 24 = - 7$ (here $C_{32} = 24$, $u_3 = 4$ and $v_2 = 13$)

	$v_1=11$	$v_2=13$	$v_3=9$	$v_4=6$
$U_1=0$	11(200)	13(50)	17	14
$U_2=5$	16	18(175)	14(125)	10(basic cell) +
$U_3=4$	21	24	13(150)	10(250)

Cij < 0 But C21 = 0 and C24 = 1, between in these values 1 is maximum number.1 ∈ C24 is called a basic solution. Next is the rule for the drawing loop. Starting from the new basic cell draw a loop in such a way that the right-angle turns are done only at the allocated cell or at the new basic c.

The rule for drawing a closed path or loop. Starting from the new basic cell draw a closed path in such a way that the right-angle turn is done only at the allocated cell and assign an alternate plus-minus sign to all the cells with right angle turn or the corner in the loop with a plus sign assigned at the new basic cell.

	$v_1=11$	$v_2=13$	$v_3=9$	$v_4=6$
$U_1=0$	11(200)	13(50)	17	14

U_2=5	16	18(175)	14(125) -	10 +
U_3=4	21	24	13(150) +	10(250) -

Consider the cells with a negative sign. Compare the allocated value (i.e., 125 and 250) and select the minimum (i.e., 125). Now subtract 125 from the cells with a minus sign and add 125 to the cells with a plus sign. And draw a new iteration. The new solution is in the below table.

11(200)	13(50)	17	14
16	18(175)	14	10(125)
21	24	13(275)	10(125)

Check the total number of allocated cells is equal to (m + n – 1). Again, find u values and v values using the formula $u_i + v_j = C_{ij}$ where C_{ij} is the cost value only for allocated cell. Assign $u_1 = 0$

$u_1 + v_1$=11, 0+ v_1=11, v_1=11(i.e. C_{11})

$u_1 + v_2$=13, 0+ v_2=13, v_2=13 (i.e. C_{12})

$u_2 + v_2$=18, u_2+13=18, u_2=5 (i.e. C_{22})

$u_2 + v_4$=10, 5+ v_4=1, v_4=5 (i.e. C_{24})

$u_3 + v_3$=13, 5+ v_3=13, u_3=8 (i.e. C_{33})

$u_3 + v_4$=10, u_3+5=10, u_3=5 (i.e. C_{34})

	v_1=11	v_2 =13	v_3 =8	v_4=5
U_1=0	11(200)	13(50)	17	14
U_2=5	16	18(175)	14	10(125)
U_3=4	21	24	13(275)	10(125)

Now, compute the non-occupied cell using the formula $P_{ij} = u_i + v_j - C_{ij}$. Let's compute this one by one.

1. For C_{13}, P_{13} = 0 + 8 -17= -9 (here C_{13} = 17, u_1 = 0 and v_3 = 8)
2. For C_{14}, P_{14} = 0 + 5 - 14 = - 9 (here C_{14} = 14, u_1 = 0 and v_4 = 5)
3. For C_{21}, P_{21} = 5 + 11 – 16 = 0 (here C_{21} = 16, u_2 = 5 and v_1 = 11)
4. For C_{23}, P_{23} = 5 + 8 – 14 = -1 (here C_{23} = 14, u_2 = 5 and v_3= 8)

5. For C_{31}, P_{31} = 5 + 11 – 21 = - 5(here C_{31} = 21, u_3 = 5 and v_1 = 11)
6. For C_{32}, P_{32} = 5 + 13 – 24 = - 6 (here C_{32} = 24, u_3 = 5 and v_2 = 13)

All $C_{ij} \leq 0$here all non-occupied cells are less than zero. So, it is optimal solution

Total cost Z=200×11 + 50×13 + 175×18 + 125×10 + 275×13 +125 ×10

Z= 12,075

The Transportation Cost using MODI Method is Rs. 12, 07, 50. 000

SIMPLEX METHOD:

The standard form of the simplex method has three main requirements one is the objective function, which should be maximization, the second is for all constraints to be linear and, the third all variables to be non-negative.

$$\mathrm{MinZ} = 11x_{A1} + 13x_{A2} + 17x_{A3} + 14x_{A4} + 16x_{B1} + 18x_{B2} + 14x_{B3} + 10x_{B4} + 21x_{C1} + 24x_{C2} + 13x_{C3} + 10x_{C4} + 0S_1 + 0S_2 + 0S_3 + 0S_4 + 0S_5 + 0S_6 + 0S_7$$

Subject to constrains.

$x_{A1}+x_{A2}+x_{A3}+x_{A4} +S_1+ 0S_2 +0S_3 +0S_4 +0S_5 +0S_6 +0S_7 = 250$

$x_{B1}+x_{B2}+x_{B3}+x_{B4} +0S_1+ S_2 +0S_3 +0S_4 +0S_5 +0S_6 +0S_7= 300$

$x_{C1}+x_{C2}+x_{C3}+x_{C4} +0S_1+ 0S_2 +S_3 +0S_4 +0S_5 +0S_6 +0S_7 = 400$

$x_{A1} + x_{B1}+ x_{C1} +0S_1+ 0S_2 +0S_3 +S_4 +0S_5 +0S_6 +0S_7 = 200$

$x_{A2} + x_{B2}+ x_{C2} +0S_1+ 0S_2 +0S_3 +0S_4 +S_5 +0S_6 +0S_{7=}$ 225

$x_{A3} + x_{B3}+ x_{C3} +0S_1+ 0S_2 +0S_3 +0S_4 +0S_5 +S_6 +0S_7 = 275$

$x_{A4} + x_{B4}+ x_{C4} +0S_1+ 0S_2 +0S_3 +0S_4 +0S_5 +0S_6 +S_7 = 250$

$x_{A1}, x_{A2}, x_{A3}, x_{A4}, x_{B1}, x_{B2}, x_{B3}, x_{B4}, x_{C1}, x_{C2}, x_{C3}, x_{C4}, S_1, S_2, S_3, S_4, S_5, S_6, S_7 \geq 0$

Since all Z_j – C_j<= 0
Hence, optimal solution is arrived with value of variables:
x_{A1}=2500, x_{A2}=22500, x_{A3}=0 x_{A4}=0 x_{B1}=17500, x_{B2}=0, x_{B3}=0, x_{B4}=12500, x_{C1}=0, x_{C2}=0, x_{C3}=27500, x_{C4}=12500

Min Z=12075
The Transportation Cost using Simplex Method is Rs. 12, 07, 50, 000.

GOAL PROGRAMMING:

Goal programming solves the set of solutions considering all problems in the given criteria giving equal importance to each goal.

$$Min\ Z = 11x_{A1} + 13x_{A2} + 17x_{A3} + 14x_{A4} + 16x_{B1} + 18x_{B2} + 14x_{B3} + 10x_{B4} + 21x_{C1} + 24x_{C2} + 13x_{C3} + 10x_{C4} + d_1^- + d_2^- + d_3^- + d_4^- + d_5^- + d_6^- + d_7^- + d_8^- + d_9^- + d_{10}^- + d_{11}^- + d_{12}^-$$

Subject to constrains

$x_{A1}+x_{A2}+x_{A3}+x_{A4} + d_1^- - d_1^+ \leq 250$

$x_{B1}+x_{B2}+x_{B3}+x_{B4} + d_2^- - d_2^+ \leq 300$

$x_{C1}+x_{C2}+x_{C3}+x_{C4}+ d_3^- - d_3^+ \leq 40$

$x_{A1} + x_{B1}+ x_{C1} + d_4^- - d_4^+ \leq 200$

$x_{A2}+ x_{B2}+ x_{C2} + d_5^- - d_5^+ \leq 225$

$x_{A3} + x_{B3}+ x_{C3} + d_6^- - d_6^+ \leq 275$

$x_{A4} + x_{B4}+ x_{C4} + d_7^- - d_7^+ \leq 25$

$x_{A1}, x_{A2}, x_{A3}, x_{A4}, x_{B1}, x_{B2}, x_{B3}, x_{B4}, x_{C1}, x_{C2}, x_{C3}, x_{C4}, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_4^+, d_5^-, d_5^+, d_6^-, d_6^+, d_7^-, d_7^+ \geq 0$

Hence, optimal solution is arrived with value of variables: x_{A1}=2500, x_{A2}=22500, x_{A3}=0 x_{A4}=0 x_{B1}=17500, x_{B2}=0, x_{B3}=0, x_{B4}=12500, x_{C1}=0, x_{C2}=0, x_{C3}=27500 x_{C4}=12500

Min Z=12075
The Transportation Cost using Goal Programming Method is Rs. 12, 07, 50, 000.

Goal programming method Z=12, 07, 50, 000

The Transportation Cost using Goal programming Method is Rs. 12, 07, 50, 000

Result and Conclusion:

Transportation problems are solved by manually and linear programming problems are solved by using linear programming solver for the result.

Methods	Optimal Solution
1.North west corner method	12,20,00,000
2.Least cost method	12,20,00,000
3.Vogels approximation method (VAM)	**12,07,50,000**
4. Modified distributed method (MODI)	**12,07,50,000**
5.Simplex method	**12,07,50,000**
6.Goal programming method	**12,07,50,000**

In this paper we analysis six methods i.e, north west corner method, least cost method, Vogel's approximation method (VAM), MODI method, simplex method, and goal programming. Here we get the least cost from Goal programming method, simplex method, Vogel's approximation method (VAM) and MODI method. But we suggest Goal programming method is best because constructing an objective function we can calculate the optimal solution by using linear programming solver software. So, we prefer Goal programming method to the user for getting low transportation cost.

References:

1. Agarana M.C, Owoloko E. A and Kolawole A. A, "Enhancing the Movement of People and Goods in a Potential World Class University Using Transportation Model", Global Journal of Pure and Applied Mathematics, Vol 12(1), pp 281-294, 2016.

2. Arthur J. L, and Lawrence K. D, "A Multiple Goal Capital Flow Model for a Chemical and Pharmaceutical Company." The Engineering Economist, Vol 30(2), pp 121-134, 1984.

3. Ashton. D. J, "Goal Programming and Intelligent Financial Simulation Models Part I-Some Problems," Accounting and Business Research, Vol 16(6), pp 3-10, 1985.

4. Charnes A, Cooper W. W and Ferguson R. O, "Optimal Estimation of Executive Compensation by Linear Programming", Management Science, Vol 1(8), pp 138-151, 1955.

5. Charnes, A., Cooper, W.W., The stepping stone method for explaining linear programming calculation in transportation problem, Management Science, Vol 1(1), pp 49 – 69, 1954.

6. Frank L. Hitchcock, "The distribution of a product from several sources to numerous localities", MIT Journal of Mathematics and Physics, Vol 20, pp 224–230,1941.

7. Kirca and Statir., "A heuristic for obtaining an initial solution for the transportation problem", Journal of operational Research Society, Vol 41, pp 865-867, 1990.

8. Kwak N.K. and Schniederjans M.J, "A goal programming model for improved transportation problem solutions", Omega, Vol 12, pp 367-370, 1979.

9. Sridevi. Polasi, Vraja Mohan Sammeta, Harish Babu G A, "Optimum Allocation of Working Hours for Plant Tissue Culture", International Journal of Mechanical and Production, Vol. 10(3), pp 13425–13434, 2020.

10. Vishwas Deep Joshi and Nilama Gupta, "Identifying more-for-less paradox in the linear fractional transportation problem using objective matrix", Mathematica, Vol 28, pp 173–180, 2012.