

One-Loop Helicity Amplitude For Light-By-Light Scattering

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Abstract

In this paper, we will aim to calculate analytically the one-loop helicity scattering amplitude and differential cross section for the process $\gamma\gamma \rightarrow \gamma\gamma$. The computation of this process takes into account non-zero mass of loop-fermion. In certain kinematic limits, compact analytic expressions exist and can be quickly obtained. We present the covariant and helicity amplitudes for this process

INTRODUCTION

This article describes the observation of the light-by-light scattering process, $\gamma\gamma \rightarrow \gamma\gamma$ in Pb+Pb collisions. Light-by-light scattering is a quantum-mechanical process. In the Standard Model, the $\gamma\gamma \rightarrow \gamma\gamma$ reaction proceeds at one-loop level at order α_{EM}^4 via virtual box diagrams involving electrically charged fermions or bosons.

Strong evidence for this process in relativistic heavy ion collisions at the Large Hadron Collider has been reported by the ATLAS [1] and CMS [2] collaborations.

In this work we describe the implementation of the SM process $\gamma\gamma \rightarrow \gamma\gamma$ through fermion and boson loops. We discuss diagrams for $\gamma\gamma \rightarrow \gamma\gamma$ process and covariant amplitude tensor structure.

$\gamma\gamma \rightarrow \gamma\gamma$ scattering is a very rare phenomenon in which two photons – particles of light – interact, producing another pair of photons. This process was among the earliest predictions of quantum electrodynamics, the quantum theory of electromagnetism, and is forbidden by classical physics theories. It is not only interesting in itself as a manifestation of an extremely rare QED phenomenon, but may be sensitive to contributions from particles beyond the Standard Model. It allows for a new generation of searches for hypothetical light and neutral particles.

Therefore, describes the observation of the light by light scattering process in Pb + Pb collisions at $\sqrt{s_{NN}} = 5.02\text{TeV}$. The investigation is conducted using a data sample corresponding to an integrated luminosity of 1.73nb^{-1} . Strong evidence for this process (59 candidate events are observed for a background expectation of 12 ± 3 events) in relativistic heavy-ion collisions at the Large Hadron Collider has been reported by the ATLAS and CMS collaborations with observed significances of 4.4 and 4.1 standard deviations, respectively. Exclusive light-by-light scattering can occur in these collisions at impact parameters larger than about twice the radius of the ions. The strong interaction becomes less significant and the electromagnetic interaction becomes more important in these ultraperipheral collision events.

Light-by-light scattering process

The covariant one-loop amplitude corresponds to a result of the straightforward standard calculation of all diagrams contributing to a given process at the one-loop level. Corresponding diagrams start from the one-loop level and in QED there are box diagrams with four internal fermions of equal mass.

Three topologies -st, su and ut channels are related by simple permutations of external photons in the diagram shown in Figure 1. At leading order, the Feynman diagrams are:

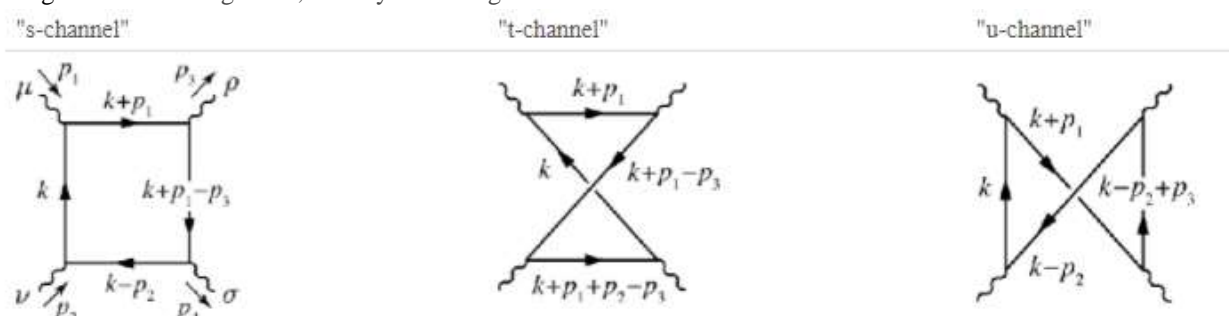


Figure 1: st-channel diagram for $\gamma\gamma \rightarrow \gamma\gamma$ process

The four moments of incoming photons are denoted by p_1 and p_2 , of the outgoing ones – by p_3 and p_4 . The four-momentum conservation law reads: $p_1 + p_2 - p_3 - p_4 = 0$

The Mandelstam variables are

$$s = (p_1 + p_2)^2 = -2p_1 p_2, \quad u = (p_1 - p_4)^2 = -2p_1 p_4, \quad t = -(p_1 - p_3)^2 = -2p_1 p_3, \quad s + t + u = 0$$

For these process, the cross section has the form:

$$d\sigma_{\gamma\gamma \rightarrow \gamma\gamma} = \frac{1}{j} |A_{\gamma\gamma \rightarrow \gamma\gamma}|^2 d\Phi^{(2)}$$

where j is the flux $A_{\gamma\gamma \rightarrow \gamma\gamma}$ is the covariant amplitude of the process:

$$d\Phi^{(2)} = (2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4) \frac{d^4 p_3 \delta(p_3^2)}{(2\pi)^3} \frac{d^4 p_4}{(2\pi)^3}$$

For the cross section one gets:

$$d\sigma_{\gamma\gamma \rightarrow \gamma\gamma} = \frac{1}{128\pi\omega^2} |A_{\gamma\gamma \rightarrow \gamma\gamma}|^2 d\cos\theta,$$

where ω is the photons energy and θ — the scattering angle in the center of mass system.

In terms of Lorenz-structures we have:

$$A_{\gamma\gamma \rightarrow \gamma\gamma} = \sum_{i=1}^{43} [F_i^{\text{bosons}}(s, t, u) + F_i^{\text{fermions}}(s, t, u)] T_i^{\alpha\beta\mu\nu}$$

The F_i are normalized by corresponding factors for fermion and boson parts.

One-loop helicity amplitude: process $\gamma\gamma \rightarrow \gamma\gamma$

In the helicity amplitudes approach we also derive tensor structure and Form Factors. The total number of HAs for this process is equal to 16. This corresponds to different combinations of external particles spin projections. In this way we can distinguish calculations of Dirac spinors.

Therefore, at this stage we observe five independent helicity amplitudes, while in the case of zero loop fermion mass one gets only four independent HAs which are very compact:

$$H_{+-+} = H_{-+-} = -1 + \frac{t-u}{s} (l_u - l_t) - \left(\frac{1}{2} - \frac{ut}{s^2}\right) (t_u l_t + \pi^2)$$

$$H_{+++} = H_{---} = -1 - i\pi \frac{t-s}{u} \left[1 + i\pi \left(\frac{t-s}{u} + i\pi \frac{t^2}{u}\right)\right] l_t l_t - \left(\frac{1}{2} - \frac{st}{u^2}\right) l_t^2$$

$$H_{-+-} = H_{+-+} = -1 - i\pi \frac{u-s}{t} \left[1 + i\pi \left(\frac{u-s}{t} + 2i\pi \frac{u^2}{t}\right)\right] l_t - \left(\frac{1}{2} - \frac{su}{t^2}\right) l_u^2$$

Matrix element squared:

$$|M|^2 = -\frac{1}{2} P_Y^\perp P_Y^\perp \text{Re} \left[e^{i(\Phi_+ - \Phi_-)} H_{++} H_{--}^* + e^{i(\Phi_+ + \Phi_-)} H_{+-} H_{-+}^* \right]$$

$$\Phi_\pm = \phi_\pm - \phi$$

$H_{--}, H_{++}, H_{-+}, H_{+-}$ are helicity amplitudes.

Finally, putting everything together we find

$$H_{-+} H_{-+}^* = \left[\frac{u-s}{t} l_u + \left(\frac{1}{2} - \frac{su}{t^2}\right) l_u^2 + \frac{t-s}{u} l_t + \left(\frac{1}{2} - \frac{s^2}{u^2}\right) l_t^2 \right]^2$$

$$+\pi^2 \left[\frac{u^2 - su - t^2 - st}{ut} + \left(\frac{u-s}{t} + 2 \left(\frac{u}{t} \right)^2 \right) l_u^2 + \left(\frac{t-s}{u} + 2 \left(\frac{t}{u} \right)^2 \right) l_t \right]^2$$

$$H_{--}H_{++}^* = \left[2 + \frac{t-u}{s} (l_u - l_t) - \left(\frac{1}{2} - \frac{tu}{s^2} \right) (l_u l_t - \pi^2) \right]^2$$

Where $H_{-+}H_{+-}^* = H_{+-}H_{-+}^*$ and $H_{--}H_{++}^* = H_{++}H_{--}^*$

Results and comparison

The differential cross section of $\gamma\gamma \rightarrow \gamma\gamma$ process in QED has a form:

$$d\sigma_{\gamma\gamma \rightarrow \gamma\gamma} = \frac{1}{128\pi\omega^2} |A_{\gamma\gamma \rightarrow \gamma\gamma}|^2 d\cos\theta,$$

Where ω is the photons frequency, θ is the scattering angle in CMS and helicity amplitudes are expressed in terms of FFs. All dependences on Mandelstam invariants and loop fermion mass, also from Passarino Veltman functions are included into these FFs.

In this paper we presented the one-loop QED correction to light-by-light scattering by fermion loops in the ultrarelativistic limit where all kinematic invariants are much greater than the relevant fermion masses. Some of the helicity amplitudes remain quite simple at one loops for tree amplitudes.

Probably the most important application would then be to compute the electroweak corrections to the W box contribution to light by light scattering processes, since that contribution dominates at high energies, where new physics contributions are most likely to be found.

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