Original Article

NEW QUADRATIC-EXPONENTIAL DISTRIBUTION

Binod Kumar Sah[1], Suresh Kumar Sahani[2*]

[1]Department of Statistics, R.R.M. Campus, Janakpurdham, Tribhuvan University, Nepal.
sah.binod01@gmail.com
[2*]Department of Mathematics, MIT Campus, Janakpurdham, Nepal.
sureshkumarsahani35@gmail.com
*Corresponding author: sureshkumarsahani35@gmail.com

DOI: 10.47750/pnr.2022.13.S04.289

Abstract

The proposed distribution is, a single parameter continuous probability distribution and has been introduced for statistical modeling of survival time data, a new form of Modified Mishra distribution with better results. It is named as 'New Quadratic-Exponential Distribution (NQED). All the required characteristics such as probability mass function, cumulative probability distribution, probability generating function and mode of NQED have been obtained. Moments about the mean and about origin have been discussed. The Hazard rate function, Reliability function and the mean residual life function of NQED have been discussed. The estimation of parameters has been discussed by the method of moments as well as the maximum likelihood method. Goodness of fit has been applied to some data-sets to test validity of the proposed distribution. The main objective to introduce the proposed distribution is to give a better alternative of Lindley distribution (LD), Mishra distribution (MD), Quadratic-Exponential distribution (QED) and Modified Mishra distribution (MMD) for statistical modeling of over-dispersed survival time data.

Keywords: Lindley distribution, Mishra distribution, Estimation of parameters, Moments and Goodness of fit

Introduction:

Our prime motive to introduce the proposed distribution is to get better alternative of LD, MD, QED and MMD by keeping equal number of parameters. All the mentioned distributions have a single parameter and so the proposed distribution. Sah [4] proposed Mishra distribution (MD) and its probability density function (pdf) has been given by

$$f_1(y) = \frac{\alpha^3}{(\alpha^2+\alpha+2)}(1+y+y^2)e^{-\alpha y} \tag{1}$$

Where, $\alpha > 0$ and $y > 0$. It was introduced in statistical literature to get a better alternative of LD, given by its pdf

$$f_2(y) = \frac{\alpha^2}{(1+\alpha)}(1+y)e^{-\alpha y} \; ; \alpha > 0, y > 0 \tag{2}$$

was obtained by Lindley [2]. Sah explained the role of MD in Poisson-Mishra distribution [5] and Generalised Poisson-Mishra distribution [6] in accident proneness. Statistics literature is full of countable and continuous probability distributions with wide range of applications in social and physical sciences. Research is an endless process because there is always some chance to add some new concepts and new visions on the previously obtained research work by others It is our responsibility to contribute to give better results and concepts than previous published works. In this process of research, Sah obtained QED [7] and pdf of which was given as

$$f_3(y) = \frac{\alpha^3}{(\alpha^3 + \alpha + 2)}(\alpha + y + y^2)e^{-\alpha y}\,;\alpha >, y > 0 \tag{3}$$

It was found that QED is a better alternative of LD as well as MD. It can be observed that both distributions have a single parameter and have been applied to the similar nature of over-dispersed data-sets. Research is an endless process and we are always keen to find a better alternative of previous one. In this way, Sah obtained Modified Mishra Distribution (MMD) [8] given by its pdf

$$f_4(y) = \frac{\alpha^3}{(\pi\alpha^2 + \alpha + 2)}(\pi + y + y^2)e^{-\alpha y}\,;\alpha >, y > 0 \tag{4}$$

It was found that MMD is a better alternative of LD, MD and QED for statistical modeling of over-dispersed survival life time data.

The proposed distribution starts with construction of theoretical frame work and end with its applications. In this process, required statistical characteristics as well as descriptive measures of Statistics of the proposed distribution have been obtained. Estimation of parameters have been discussed. Nature of the proposed distribution has been discussed on the basis of dispersion, skewness and kurtosis. The hazard rate function, the mean residual life time and reliability function have been discussed. Goodness of test has been applied to some previously used data by others.

Material and Methods:

The proposed distribution is based on the core concept of continuous probability distribution which contains a single parameter. It starts with the construction of theoretical frame work such as probability density function, cumulative probability distribution function, moment generating function and mode. Moment about the mean as well as about origin have been obtained. Parameter of NQED has been estimated by the method of moments and the maximum likelihood method. Nature of the distribution has been discussed according to dispersion, skewness and kurtosis. Goodness of fit has been applied to some over-dispersed data-sets to test validity of the theoretical work and ends with the concluding remarks about NQED.

Results:

The results obtained about NQED are classified under following sub-headings.

- New Quadratic-Exponential Distribution (NQED) and Related Characteristics

- Moments, Skewness and Kurtosis of NQED

- Estimation of Parameters of NQED

-The Hazard Rate Function, Reliability Function and The Mean Residual Life function of NQED, and

- Applications of NQED.

New Quadratic-Exponential Distribution (NQED) and Related Characteristics:

We propose a new structure of MMD and it is named as New Quadratic-Exponential distribution (NQED). It is a single parameter continuous probability distribution which follows all the basic characteristics of continuous probability distribution. Let us consider a random variable Y having a single parameter α ,then pdf of NQED is given by

$$f(y)=\frac{\alpha^3}{(\pi\alpha^2+2)}(\pi+y^2)e^{-\alpha y} \quad (5)$$

where $y>0$ and $\alpha>0$ .The expression (5) is the pdf of NQED.

Cumulative distribution function (cdf) of the NQED (5) can be obtained as

$$F(Y)=P(Y\le y)=\int_0^y f(y)dy=\frac{\alpha^3}{(\pi\alpha^2+2)}\int_0^y(\pi+y^2)e^{-\alpha y}\,dy$$

$$=1-e^{-\alpha y}-\frac{\alpha y(2+\alpha y)}{(2+\pi\alpha^2)}e^{-\alpha y} \quad (6)$$

The expression (6) is the cdf of the NQED (5). Graphical representations of pdf and cdf for varying values of parameter of NQED (5) are given below.

Fig1: Graph of pdf of NQED at $\alpha=0.2,0.4,0.6,0.8$

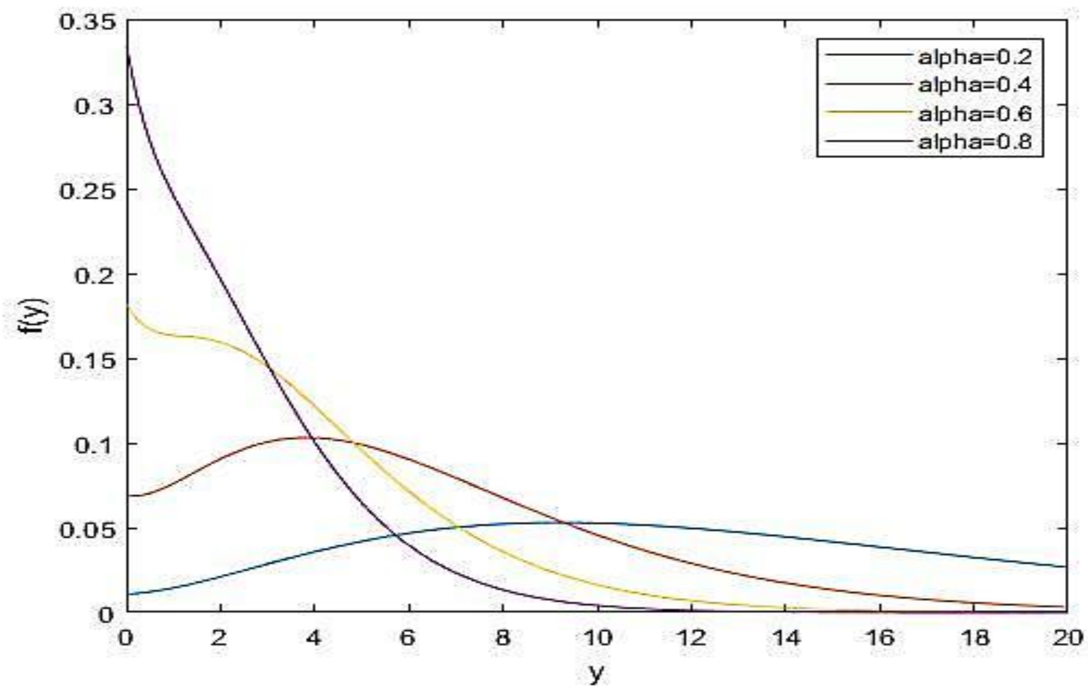


Fig2: Graph of pdf of NQED at $\alpha = 0.5, 1.0, 1.5, 2.0$

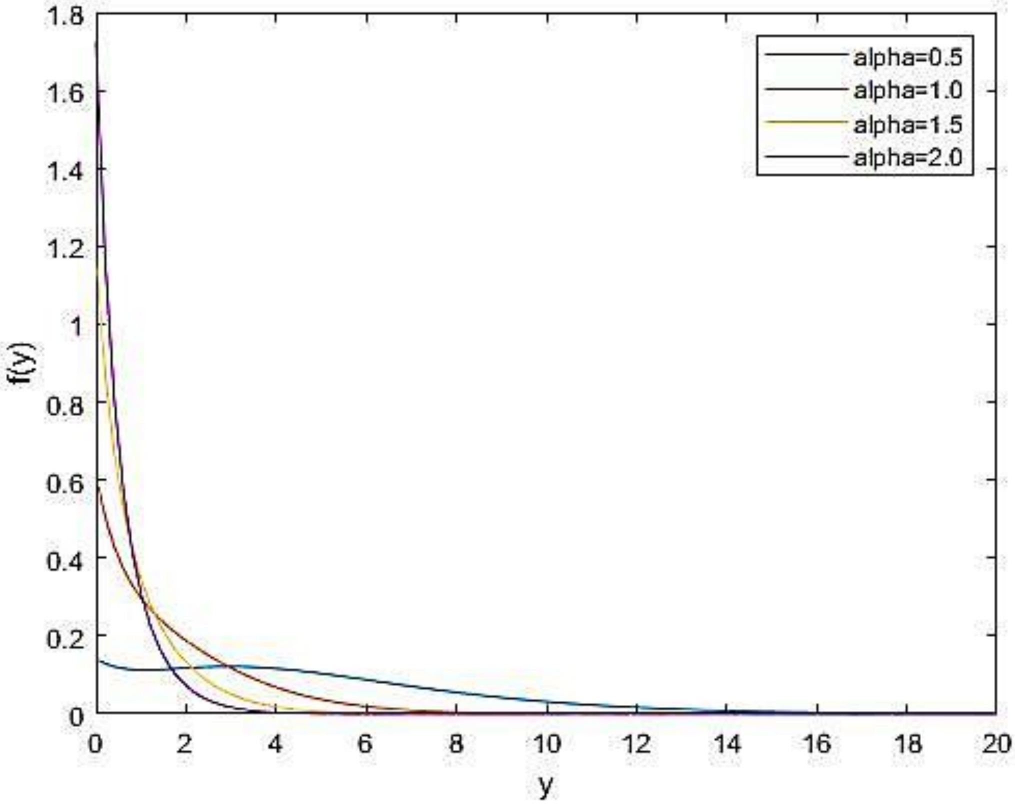


Fig.3: Graph of cdf of NQED at $\alpha = 0.2, 0.4, 0.6, 0.8$

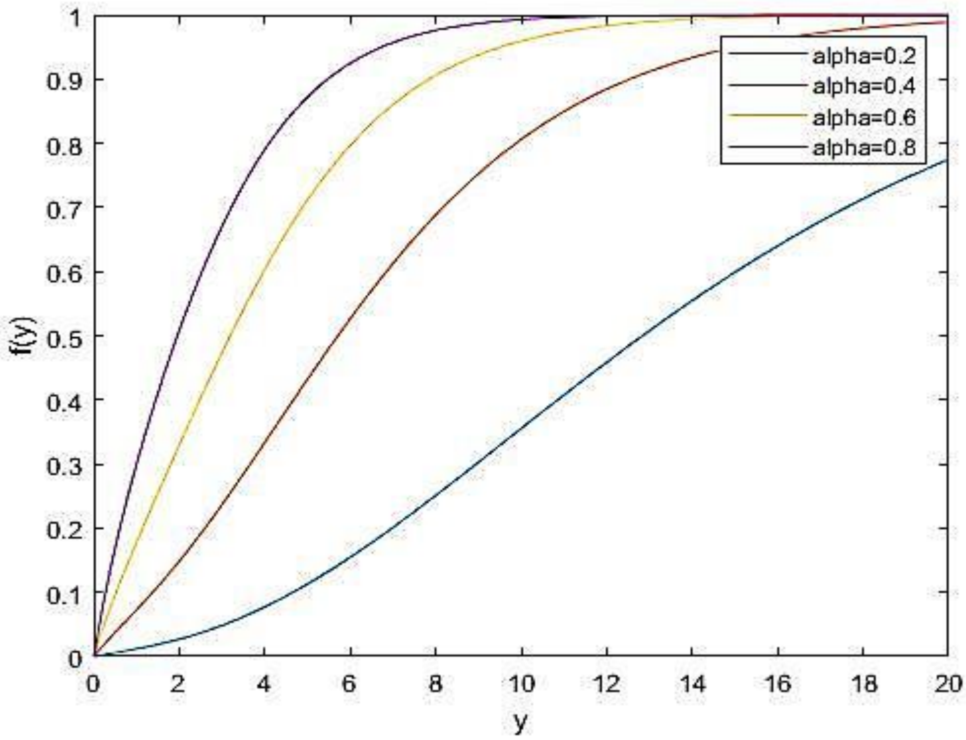
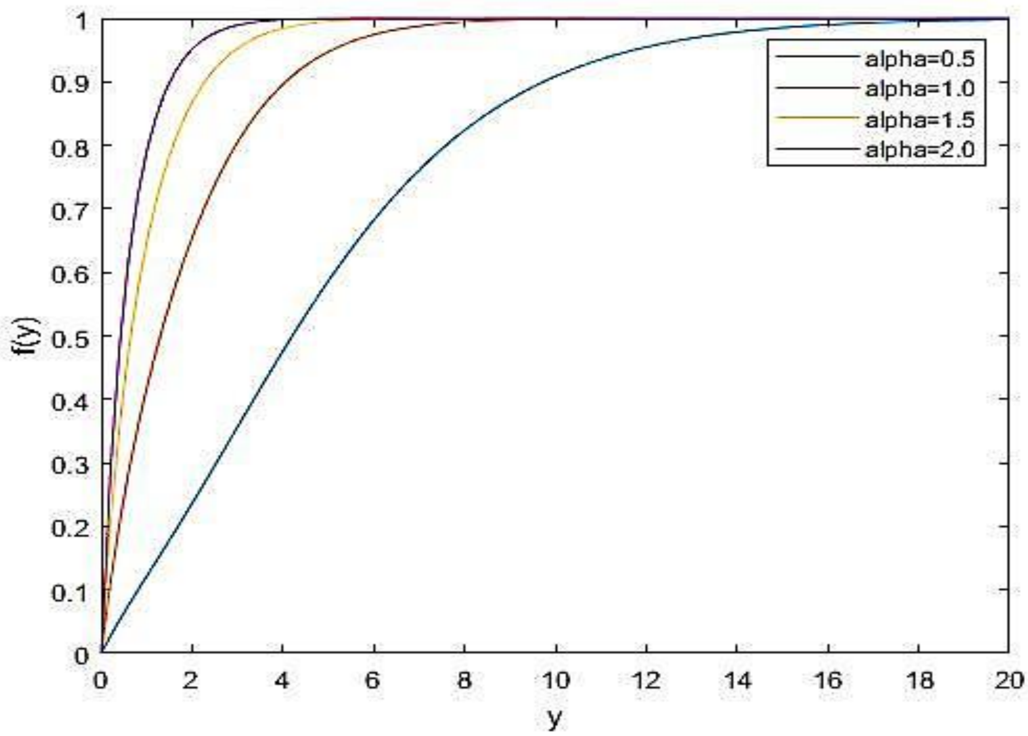


Fig.4: Graph of cdf of NQED at $\alpha = 0.5, 1.0, 1.5, 2.0$



Mode of NQED:

Mode of NQED is the value of random variable Y for which pdf is maximum. Using the rule of maxima, the following condition should be satisfied (i) $f'(y)=0$ and (ii) $f''(y)<0$. At first, differentiate the expression (5) with respect to y, we get

$$\frac{d}{dy}[f(y)] = \frac{\alpha^3}{(\pi\alpha^2+2)}\frac{d}{dy}[(\pi+y^2)e^{-\alpha y}] = \frac{\alpha^3}{(\pi\alpha^2+2)}\left[(\pi+y^2)(-\alpha e^{-\alpha y})+e^{-\alpha y}(2y)\right] \quad (7)$$

Applying $f'(y)=0$, we get

$$\left[(\pi+y^2)(-\alpha e^{-\alpha y})+e^{-\alpha y}(2y)\right]=0$$

Or, $\alpha y^2 - 2y + \pi\alpha = 0$ (8)

$$y = \frac{1 \pm \sqrt{\left[(1-\pi\alpha^2)\right]}}{\alpha} \quad (9)$$

The expression (9) will be mode of NQED (5) if $f''(y)<0$.The expression (9) will exist if $(1-\pi\alpha^2)>0$.That is, the mode of NQED will be exist if $0<\alpha<0.564189581$.

Differentiate the expression (7) with respect to y, we get

$$\frac{d}{dy}\left[\frac{d[f(y)]}{dy}\right] = \frac{d}{dy}\left[\frac{\alpha^3}{(\pi\alpha^2+2)}\left[(\pi+y^2)(-\alpha e^{-\alpha y})+e^{-\alpha y}(2y)\right]\right] \quad (10)$$

$$= \frac{\alpha^3}{(\pi\alpha^2+2)}(\pi\alpha^2+\alpha^2 y^2-4\alpha y+2)e^{-\alpha y}$$

Applying $f''(y)<0$, we get

$(\pi\alpha^2+\alpha^2 y^2-4\alpha y+2)<0$ if $0<\alpha<0.588073$ for $y=\frac{1\pm\sqrt{[(1-\pi\alpha^2)]}}{\alpha}$. Hence, mode of NQED will be

$$y=\frac{1\pm\sqrt{[(1-\pi\alpha^2)]}}{\alpha}.$$

Moment Generating Function [$M_Y(t)$]: It is obtained as

$$M_Y(t)=\int_0^\infty e^{ty} f(y)dy=\frac{\alpha^3}{(\pi\alpha^2+2)}\int_0^\infty e^{ty}(\pi+y^2)e^{-\alpha y}dy$$

$$=\frac{\alpha^3}{(\pi\alpha^2+2)}\int_0^\infty(\pi+y^2)e^{-(\alpha-t)y}dy=\frac{\alpha^3}{(\pi\alpha^2+2)}\left[\pi\int_0^\infty e^{-(\alpha-t)y}dy+\int_0^\infty y^2 e^{-(\alpha-t)y}dy\right]$$

$$=\frac{\alpha^3}{(2+\pi\alpha^2)}\left[\frac{\pi}{(\alpha-t)}+\frac{2}{(\alpha-t)^3}\right]=\frac{\alpha^3}{(2+\pi\alpha^2)}\left[\frac{2+\pi(\alpha-t)^2}{(\alpha-t)^3}\right] \tag{11}$$

The expression (11) is the M.G.F. of NQED (5).

Moments of NQED:

Statistical moments are capable of studying descriptive statistics. In this section, we obtain

-The rth moment about origin

-The first four moment about the mean and

-Dispersion, Skewness and Kurtosis of the NQED.

The rth moment about origin: It is obtained as

$$\mu_r'=E[Y^r]=\int_0^\infty y^r f(y)dy=\frac{\alpha^3}{(\pi\alpha^2+2)}\left[\int_0^\infty y^r(\pi+y^2)e^{-\alpha y}dy\right] \tag{12}$$

$$=\frac{\alpha^3}{(\pi\alpha^2+2)}\left[\pi\int_0^\infty y^r e^{-\alpha y}dy+\int_0^\infty y^{r+2}e^{-\alpha y}dy\right]=\frac{\alpha^3}{(\pi\alpha^2+2)}\left[\frac{\pi\Gamma(r+1)}{\alpha^{(r+1)}}+\frac{\pi\Gamma(r+3)}{\alpha^{(r+3)}}\right]$$

After a little simplification, we get

$$\mu_r'=\frac{r!}{\alpha^r}\left[\frac{\{\pi\alpha^2+(r+2)(r+1\}}{(2+\pi\alpha^2)}\right] \tag{13}$$

The expression (13) is the rth moment about origin of NQED (5). Substituting the value of $r=1,2,3,4$ in the expression (13), the first four moments about origin have been obtained as

$$\mu_1'==\frac{1}{\alpha}\left[\frac{(\pi\alpha^2+6)}{(2+\pi\alpha^2)}\right] \tag{14}$$

$$\mu_2' == \frac{2!}{\alpha^2}\left[\frac{(\pi\alpha^2+12)}{(2+\pi\alpha^2)}\right] \tag{15}$$

$$\mu_3' == \frac{3!}{\alpha^3}\left[\frac{(\pi\alpha^2+20)}{(2+\pi\alpha^2)}\right] \tag{16}$$

$$\mu_4' == \frac{4!}{\alpha^4}\left[\frac{(\pi\alpha^2+30)}{(2+\pi\alpha^2)}\right] \tag{17}$$

Central Moments of NQED:

The first four central moments of NQED (5) have been obtained as

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - (\mu_1')^2 = \frac{2!}{\alpha^2}\left[\frac{(\pi\alpha^2+12)}{(2+\pi\alpha^2)}\right] - \left[\frac{1}{\alpha}\frac{(\pi\alpha^2+6)}{(2+\pi\alpha^2)}\right]^2 = \left[\frac{\{(\pi^2\alpha^4+16\pi\alpha^2+12)\}}{\{\alpha(2+\pi\alpha^2)\}^2}\right] \tag{18}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$

$$= \frac{3!}{\alpha^3}\left[\frac{(\pi\alpha^2+20)}{(2+\pi\alpha^2)}\right] - 3\left[\frac{2!}{\alpha^2}\left[\frac{(\pi\alpha^2+12)}{(2+\pi\alpha^2)}\right]\right]\left[\frac{1}{\alpha}\frac{(\pi\alpha^2+6)}{(2+\pi\alpha^2)}\right] + 2\left[\frac{1}{\alpha}\frac{(\pi\alpha^2+6)}{(2+\pi\alpha^2)}\right]^3$$

$$= \left[\frac{\{6(\pi\alpha^2+20)(2+\pi\alpha^2)^2 - 6(\pi\alpha^2+12)(\pi\alpha^2+6)(2+\pi\alpha^2) + 2(\pi\alpha^2+6)^3\}}{\{\alpha(2+\pi\alpha^2)\}^3}\right]$$

$$= \frac{2(\pi\alpha^2)^3 + 60(\pi\alpha^2)^2 + 72(\pi\alpha^2) + 48}{\left[\alpha(\pi\alpha^2+2)\right]} \tag{19}$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$

$$= \frac{4!}{\alpha^4}\left[\frac{(\pi\alpha^2+30)}{(2+\pi\alpha^2)}\right] - 4\left[\left\{\frac{3!}{\alpha^3}\frac{(\pi\alpha^2+20)}{(2+\pi\alpha^2)}\right\}\left\{\frac{1}{\alpha}\frac{(\pi\alpha^2+6)}{(2+\pi\alpha^2)}\right\}\right]$$

$$+6\left[\left\{\frac{2!}{\alpha^2}\left[\frac{(\pi\alpha^2+12)}{(2+\pi\alpha^2)}\right]\right\}\left\{\frac{1}{\alpha}\frac{(\pi\alpha^2+6)}{(2+\pi\alpha^2)}\right\}^2\right] - 3\left[\frac{1}{\alpha}\frac{(\pi\alpha^2+6)}{(2+\pi\alpha^2)}\right]^4$$

$$= \left[\frac{\left\{\begin{array}{l}24(\pi\alpha^2+30)(2+\pi\alpha^2)^3 - 24(\pi\alpha^2+20)(\pi\alpha^2+6)(2+\pi\alpha^2)^2 \\ +12(\pi\alpha^2+12)(\pi\alpha^2+6)^2(2+\pi\alpha^2) - 3(\pi\alpha^2+6)^4\end{array}\right\}}{\{\alpha(2+\pi\alpha^2)\}^4}\right]$$

$$= \frac{\left[9(\alpha\pi^2)^4 + 384(\alpha\pi^2)^3 + 1224(\alpha\pi^2)^2 + 1724(\alpha\pi^2) + 720\right]}{\left[\alpha(\pi\alpha^2+2\right]^4} \tag{20}$$

Nature of NQED (5) according to dispersion: Index of dispersion (I) is defined as

$$I = \frac{Variance}{Mean} = \frac{(\pi^2\alpha^4+16\pi\alpha^2+12)}{[\alpha(\pi\alpha^2+2)(\pi\alpha^2+6)]} \tag{21}$$

If ‘I’ is equal to one, then we get a fifth-degree polynomial equation in terms of α as follows

$$f(\alpha)=(\pi^2\alpha^4+16\pi\alpha^2+12)-(\pi^2\alpha^5+8\pi\alpha^3+12\alpha)=0 \quad (22)$$

Which may be solved by using Newton-Rapson method or Regula-Falsi method. At $\alpha=1.480997$, $f(\alpha)=0$ .Hence, NQED will be equi-dispersed if $\alpha=1.480997$. It will be over-dispersed if $\alpha<1.480997$ and it will be under dispersed if $\alpha>1.480997$.

$$C.V.=\left(\frac{\sigma_Y}{\mu_1'}\right)100=\left[\frac{\sqrt{(\pi^2\alpha^4+16\pi\alpha^2+12)}}{[\alpha(\pi\alpha^2+2)(\pi\alpha^2+6)]}\right]100 \quad (23)$$

The expression (23) is the co-efficient of variation of NQED (5).

Nature of NQED according to skewness: Co-efficient of skewness is obtained as follows.

$$\gamma_1=\frac{\mu_3}{\mu_2^{3/2}}=\frac{[2(\pi\alpha^2)^3+60(\pi\alpha^2)^2+72(\pi\alpha^2)+48]}{(\pi^2\alpha^4+16\pi\alpha^2+12)^{3/2}} \quad (24)$$

It is found that $(2/\sqrt{3})<\gamma_1<\infty$. Hence, NQED (5) is positively skewed in shape.

Nature of NQED according to kurtosis: Co-efficient of kurtosis is obtained as

$$\beta_2=\frac{\mu_4}{\mu_2^2}=\frac{\left[9(\alpha\pi^2)^4+384(\alpha\pi^2)^3+1224(\alpha\pi^2)^2+1724(\alpha\pi^2)+720\right]}{(\pi^2\alpha^4+16\pi\alpha^2+12)^2} \quad (25)$$

It is found that $5<\beta_2<\infty$ and hence NQED (5) is leptokurtic in size.

Estimation of Parameters of NQED:

Here, only two methods of moments have been discussed to estimate the parameter (α) of NQED (5).

(a) *The Method of Moments*:

An estimate of α can be obtained by using the first moment about origin of NQED (5). We get

$$\mu'\alpha(2+\pi\alpha^2)-(\pi\alpha^2+6)=0 \quad (26)$$

The expression (26) is the third-degree polynomial equation in α . Replacing the population mean by the sample mean and solving the expression (26) by the Newton-Rapson or Regula-Falsi method, we get an estimate of α .

(b) *The Method of Maximum Likelihood*:

Let ($(y_1, y_2, ..., y_n)$ be a sample of size n drawn from a population of size N which follows NQED (5). The likelihood function of NQED (5) can be obtained as

$$L=\prod_{i=1}^{n} f(y;\alpha)=\left(\frac{\alpha^3}{(\pi\alpha^2+2)}\right)^n\left[\prod_{i=1}^{n}(\pi+y^2)\right]e^{-n\alpha\bar{y}}$$

(27)

The log likelihood equation is

Or, $\ln L = n\ln(\alpha)^3 - n\ln(2+\pi\alpha^2) + \sum_{i=1}^{n} \ln(\pi + y_i^2) - n\alpha\bar{y}$

Differentiation of the log likelihood equation with respect to α , we get

Or, $\frac{\partial \ln L}{\partial \alpha} = \frac{3n}{\alpha} - \frac{n(2\pi\alpha)}{(2+\pi\alpha^2)} - n\bar{y} = 0$

Or, $\bar{y} == \frac{1}{\alpha}\left[\frac{(\pi\alpha^2+6)}{(2+\pi\alpha^2)}\right]$ (28)

Or, $\bar{y}\alpha(2+\pi\alpha^2) - (\pi\alpha^2+6) = 0$ (29)

The expression (28) is the mean of NQED. Putting the value of the sample mean in the expression (29) and solving the equation for α ,we get an estimate of α .

***The Reliability Function, Hazard Rate Function and Mean Residual Life Function of NQED*:**

The Reliability Function:

It is widely used in manufacturing sector to fix a guaranty or warranty period of products because it measures probability of survival lifetime without failure in a certain interval of time or we can say that the probability of time taken by a product for first failure. Let Y denotes a continuous random variable (r.v.) of survival lifetime of a component and the probability that component will survive until time ‘t’ is called reliability function denoted by R(t) and it is given by

$$R(t) = P(Y > t) = \int_t^{\infty} f(y)dy = 1 - F(t)$$

$$= \frac{\left[(2+\pi\alpha^2) + \alpha t(2+\alpha t)\right]}{(2+\pi\alpha^2)} e^{-\alpha t} \tag{30}$$

At $t=0$, $R(t)=1$ then the component of the system is said to be perfect reliable to work properly. At $t=\infty, \lim_{t\to\infty}[R(t)] = 0$. Hence, we can say that the value of reliability function decreases as working time of the system increases and vice-versa. i.e., $R(t) \propto 1/t$.

Fig.5. Graph of R(t) of NQED at $\alpha = 0.2, 0.4, 0.6, 0.8$

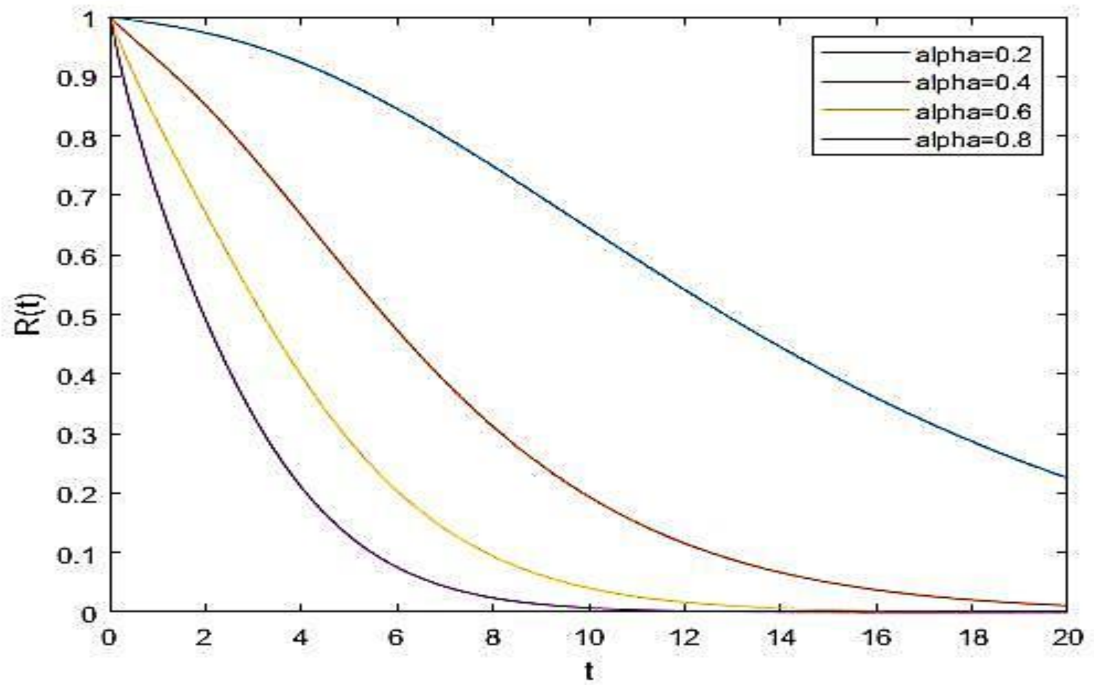
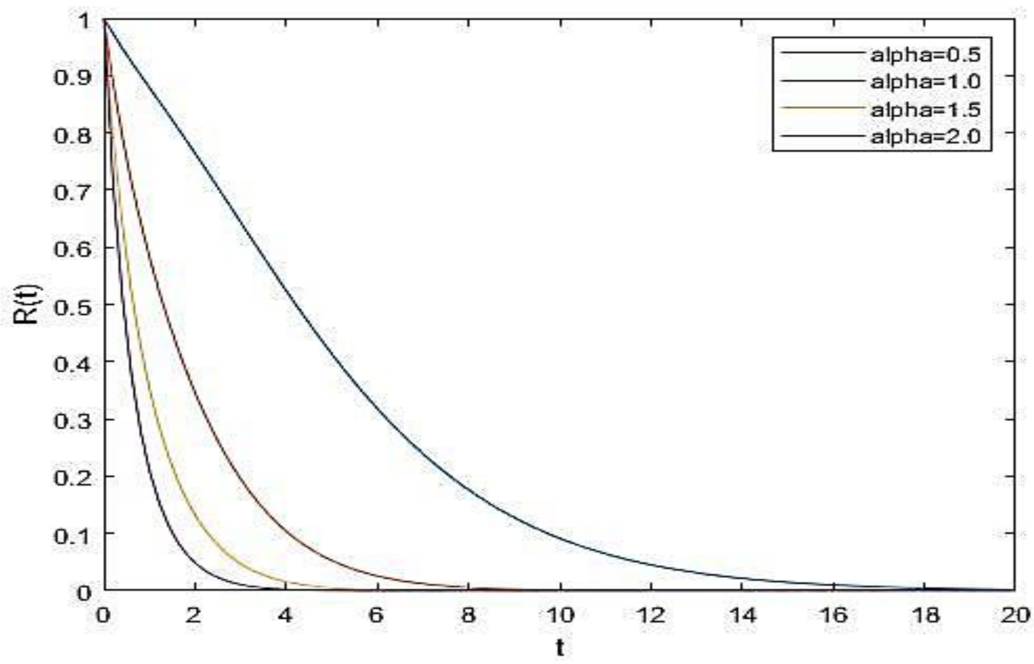


Fig.6. Graph of R(t) of NQED at $\alpha = 0.5, 1.0, 1.5, 2.0$



***The Hazard Rate Function**:*

It is defined as the ratio of probability density function to the reliability function.

$$h(y)=\lim_{\Delta y\to 0}\frac{P(Y<y+\Delta y/Y>y)}{\Delta y}=\frac{f(y)}{1-F(y)} \tag{31}$$

$$h(y)=\frac{f(y)}{R(t)}=\frac{\alpha^3(\pi+y^2)}{(2+\pi\alpha^2)+\alpha y(2+\alpha y)} \tag{32}$$

The failure rate function of NQED (5) until time 't' is thus obtained as

$$h(y=t)=\frac{\alpha^3(\pi+t^2)}{(2+\pi\alpha^2)+\alpha t(2+\alpha t)} \tag{33}$$

At $t=0$, $$h(y=t=0)=\frac{\pi\alpha^3}{(2+\pi\alpha^2)}>0 \tag{34}$$

It is also obvious that h(y) is an increasing function of y and α .

Mean Residual Life Function

In reliability studies, the expected additional life time given that a component has survived until time 't' is called mean residual life time. Let a random variable Y denotes the life of a component, the mean residual life function is given by

$$m(y)=E\left[Y-y/Y>y\right]=\frac{\int_y^\infty\left[1-F(t)\right]dt}{1-F(y)} \tag{35}$$

To obtain *m(y)* of the NQED (5), we have to calculate the following measures

$$F(Y)=P(Y\le y)=\int_0^y f(y)dy=\frac{\alpha^3}{(\pi\alpha^2+2)}\int_0^y(\pi+y^2)e^{-\alpha y}\,dy\ ;\ \text{x}>0,\ \phi>0$$

$$=1-e^{-\alpha y}-\frac{\alpha y(2+\alpha y)}{(2+\pi\alpha^2)}e^{-\alpha y} \tag{36}$$

and

Where *f*(*y*) and *F*(*y*) are the probability density function and probability distribution function of NQED (5).

$$1-F(y)=\frac{\left[(2+\pi\alpha^2)+\alpha y(2+\alpha y)\right]}{(2+\pi\alpha^2)}e^{-\alpha y}$$

$$1-F(t)=e^{-\alpha t}+\frac{\alpha t(2+\alpha t)}{(2+\pi\alpha^2)}e^{-\alpha t} \tag{37}$$

$$\int_y^\infty\{1-F(t)\}dt=\int_y^\infty\left[e^{-\alpha t}+\frac{\alpha t(2+\alpha+\alpha t)}{(2+\alpha+\pi\alpha^2)}e^{-\alpha t}\right]dt$$

$$=\left[\frac{(2+\pi\alpha^2)+(4+4\alpha y++\alpha^2y^2)}{\alpha(2+\pi\alpha^2)}\right]e^{-\alpha y} \tag{38}$$

Putting the value of $\int_{y}^{\infty}\left[1-F(t)\right]dt$ and $1-F(y)$ in equation (35), the mean residual life function has been obtained as

$$m(y)=\frac{(2+\pi\alpha^2)+(4+4\alpha y++\alpha^2 y^2)}{\alpha\left[(2+\pi\alpha^2)+\alpha y(2+\alpha y)\right]} \tag{39}$$

$$\text{At } y=0\,,\; m(y=0)=\frac{(6+\pi\alpha^2)}{\alpha(2+\pi\alpha^2)}=\mu_1' \tag{40}$$

It can also be seen that at $y=0$,the mean residual life function is the mean of NQED (5). It can also be seen that $m(y)$ is a decreasing function of y and α .

Applications of NQED:

To test validity of the theoretical work, the fitting of NQED (5) has been applied to the following to data.

Example (1): Survival times (in days) of guinea pigs infected with virulent tubercle bacilli, reported by Bjerkedal [1].

Survival Time (In days)	0 -80	80-160	160-240	240-320	320-400	400-480	480-560
Observed frequency	8	30	18	8	4	3	1

Example (2): Mortality grouped data for blackbirds species reported by Paranjpe and Rajarshi [3]

Survival Time (In days)	0 -1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	>8
Observed frequency	192	60	50	20	12	7	6	3	2

The expected frequencies according to the LD, MD, QED and MMD have also been given, for ready comparison with those obtained by the NQED, in the following tables.

Table I: Expected V Observed of Example (1)

Survival Time (In days)	**Observed frequency**	**Expected frequency**				
		Lindley	**Mishra**	**QED**	**MMD**	**NQED**
0 – 80	8	16.1	10.8	10.8	10.8	10.7
80 – 160	30	21.9	24.8	24.8	24.8	24.9
160 – 240	18	15.4	19.0	19.0	19.0	19.0
240 – 320	8	9.0	10.1	10.1	10.1	10.1
320 – 400	4	5.5	4.5	4.5	4.5	4.5

400 – 480	3	1.8	1.8	1.8	1.8	1.8
480 -560	1	2.3	1.0	1.0	1.0	1.0
Total	72	72.0	72.0	72.0	72.0	72.0
$\hat{\alpha}$		0.011	0.016431	0.016519	0.016514	0.01655966
$\chi^2\,(df)$		7.7712(3)	1.98(3)	1.98(3)	1.98(3)	1.89(3)
μ_1' =181.11111		μ_2' =43911.11111				

Table II: Expected V Observed Frequencies of Example (2)

Survival Time (In Days)	**Observed frequency**	**Expected frequency**				
		LD	**MD**	**QED**	**MMD**	**NQED**
0-1	192	173.5	142.8	145.9	165.0	163.2
1-2	60	98.6	104.5	101.2	83.3	84.8
2-3	50	46.5	58.7	57.6	51.9	49.7
3-4	20	20.1	27.5	27.5	27.4	27.6
4-5	12	8.1	11.5	12.0	13.4	14.2
5-6	7	3.2	4.5	4.8	6.2	6.9
6-7	6	1.4	1.6	1.8	2.8	3.2
7-8	3	0.4	0.5	0.7	1.2	1.4
>8	2	0.3	0.4	0.5	0.8	1.0
Total	352	352.0	352.0	352.0	352.0	352.0
$\hat{\alpha}$		.984	1.311017	1.269721	1.10616888	1.085198846
χ^2 (df)		49.85(4)	46.37(4)	39.64(4)	17.60(5)	17.19(5)
μ_1' *=1.56818*		μ_2' *=5.005682*				

From table(I) and (II), we can observe that the value of Chi-square of NQED is less than the LD, MD, QED and MMD.

Concluding Remarks:

In this paper, several structural properties such as probability density function, cumulative distribution function, moment generating function and mode have been obtained. The moments about origin as well as the mean have been derived. Parameter of this distribution has been estimated by the method of moments and the maximum likelihood. The reliability function, hazard rate function and mean residual life function have been obtained and discussed. The highlighted remarks about the NQED (5) are

* It has been observed that it will be Equi-dispersed when $\alpha = 1.480997$,Over-dispersed when $\alpha < 1.480997$ and Under-dispersed when $\alpha > 1.480997$.

* It has been observed that $(2/\sqrt{3}) < \gamma_1 < \infty$. Hence, it is positively skewed in shape.

* It has been obtained that $5 < \beta_2 < \infty$ and hence it is leptokurtic in size.

* From tables I and II, it has been observed that NQED (5) gives a better fit to the similar data-sets alternative of than LD [2], MD [4], QED [7] and MMD [8] under statistical homogeneity.

Conflict of Interest:

The authors have declared that there is no conflict of interest.

Acknowledgement:

The authors express their gratitude to the referee and entire team for his valuable comments and suggestions which help to improve the quality of the research article.

References:

1. T. Bzerkedal. Acquisition of Resistance in Guinea Pigs Infected with Different Doses of Virulent Tubercle Bacilli, American Journal of Epidemiology, 72 (1) (1960) ,130 – 148.
2. D.V. Lindley. Fiducial distributions and Bayes theorem, *Journal of the Royal Statistical Society*, *Ser. B*, 20(1958), 102- 107.
3. S. Paranjpe and M.B. Rajarshi. Modeling Non-Monotonic Survivorship Data with Bath Tube Distribution, Ecology, 67(6) (1986), 1693-1695. doi:10.2307/1939102
4. B.K. Sah. Mishra Distribution, *International Journal of Mathematics and Statistics Invention*, 3(8) (2015), 14-17.
5. B.K. Sah. Poisson-Mishra Distribution, *International Journal of Mathematics and Statistics Invention*, 5(3) (2017), 25-30.
6. B.K. Sah. A Generalised Poisson-Mishra Distribution. *Nep.J. Stat.*, 2(2018), 27-36. doi: http://dx.doi.org/10.3126/njs.v2i0.21153
7. B.K. Sah. Quadratic-exponential distribution, *The Mathematics Education*, LVI (1) (2022), 01-17. doi: https://doi.org/10.5281/zenodo.6381529
8. B.K. Sah. Modified Mishra distribution, *The Mathematics Education*, LVI (2) (2022), 01-17.