Original Article

A Multi Server Fuzzy Feedback Queuing Model With Encouraged Arrivals, Revoked And Retaining Back Out Customers

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Abstract

Fuzzy number grading is actually indispensable in Artificial Intelligence, Data Mining, Decision Analysis and Optimization, among other applications. This paper proposed Wingspan's method for ranking triangular fuzzy numbers pertains to the region lying between the slope of the membership function and the horizontal real axis. In a fuzzy atmosphere, we discover performance measurements of promoted arrivals, revoking and keeping clients who repudiate. This strategy clearly makes evident its superiority apropos of being accommodating and effective. The numerical example for a triangular fuzzy number is competently exhibited.

Keywords Fuzzy set, Triangular fuzzy number, M/M/c/N queue, Encouraged arrivals, Revoking, Retained customers, Wingspans fuzzy ranking, Performance measures

1. Introduction

Queuing Theory is concerned with the statistical description of the behaviour of queues. In a queuing system, feedback customer refers to a situation wherein a customer is discontented with a service. It is very often observed that, the clients log off from the service to the system for a long time before the receipt of the service.

In Today's fast-paced business world, to retain existing customers and to boost them up so as to bring in new customers is a herculean task. Discounts and concessions attract customers towards the particular firm up to an extent. Those attracted customers are termed as encouraged arrivals. R. Kumar and S.K. Sharma [12] have proposed feedback queue model as a way to keep up the backing out clients. Through a thorough investigation, B.K. Som and S. Seth devised [1] the system of queues in order to encourage the arrivals.

A large focus of the queue system have been investigated by notable investigators like R.J. Li and E.S. Lee [14], R.S. Negi and E.S. Lee, S.P. Chen, J.J. Buckely as in a fuzzy environment. Ranking fuzzy numbers have been analysed by numerous researchers, like Jain, Yager, Cheng [3], Wang and Lee [18]. Wingspans fuzzy ranking method was proposed by Westman and Wang [17].

In this paper, we track down the performance measures of encouraged arrivals, revoking and retaining customers who back out in multi-server fuzzy feedback queuing model using Wingspans fuzzy ranking method. This method is in fact very simple to compute the actual crisp values of the given queuing model.

2. Preliminaries

Definition: 2.1

C is a fuzzy set defined on U and can be written as a collection of ordered pairs if U is a universe of discourse and x is a particular element of U.

$$\tilde{C} = \{(x, \phi_{\tilde{C}}(x)) / x \in U\}$$

Definition: 2.2

A fuzzy set $\tilde{C}$ defined on the universe set U is said to be normal if and only if $\underset{x \in U}{Sup}\ \phi_{\tilde{C}}(x) = 1.$

Definition: 2.3

A fuzzy set $\tilde{C}$ defined on the universe set U is said to be Convex if and only if, $\phi_{\tilde{C}}(\lambda x + (1-\lambda)y) \geq \min\{\phi_{\tilde{C}}(x), \phi_{\tilde{C}}(y)\} \forall x, y \in U \text{ and } \lambda \in [0,1]$.

Definition: 2.4

A fuzzy set $\tilde{C}$ in the universe of discourse U is a fuzzy number if it fulfils the following conditions:

(i) $U = R$;

(ii) $\tilde{C}$ is Normal;

(iii) $\tilde{C}$ is Convex;

(iv) The membership function $\phi_{\tilde{C}}$ is piecewise continuous;

(v) There exists one and only one $x \in R$, such that: $\phi_{\tilde{C}}(x) = 1.$

Definition: 2.5

A fuzzy number $\tilde{C}$ is called triangular fuzzy number if its membership function is given by:

$$\phi_{\tilde{C}}(x) = \begin{cases} \dfrac{(x-a)}{(b-a)} & , \ a \leq x \leq b \\ 1 & , \ x = b \\ \dfrac{(x-c)}{(b-c)} & , \ b \leq x \leq c \\ 0 & , \ otherwise \end{cases}$$

Denoted by $\tilde{C}$ = (a / b / c) or $\tilde{C}$ = (a, b, c)

3. Mathematical Model Formulation

3.1. Assumptions

Based on the assumptions stated below, encouraged arrivals, revoking and retaining back out customers in multi-server feedback model of a fuzzy queue are being derived:

(i) The occurrence befall successively in line with Poisson distribution with the fuzzy rate $\tilde{\lambda}(1+\eta)$, in which $'\eta'$ symbolizes the encouraging factor. It depicts the proportional increase in the customers' rate of arrival and computed from previous or noticed data. We take η = 50 percent.

(ii) Service time follows an exponential distribution with fuzzy rate $\tilde{\mu}$.

(iii) The Queue discipline is First Come, First Served (FCFS).

(iv) Service is provided through a Multi-Channel.

(v) The system capacity is finite and represented as N.

(vi) The reneging time (revoking time) is distributed independently and identically for parameter $\tilde{\xi}$.

(vii) It is certainly feasible that a dissatisfied customer would rejoin the queue as a feedback customer for the fulfilment of service with probability q and would abandon the queue acceptably with probability p = 1 – q.

(viii) Retaining back out customer probability will be q', and the probability of not retaining back out customer is $p' = 1 - q'$.

3.2. Steady – State Solution

The steady-state probabilities equations governing the fuzzy queuing model are derived as follows:

$$\tilde{\mu} p P_1 = \tilde{\lambda}(1+\eta) P_0 \quad ; n = 0 \qquad \text{... (1)}$$

$$(n+1)\tilde{\mu} p P_{n+1} = \left\{\tilde{\lambda}(1+\eta) + n\tilde{\mu} p\right\} P_n - \tilde{\lambda}(1+\eta) P_{n-1} \quad ; 1 \le n \le c-1 \qquad \text{... (2)}$$

$$\left\{c\tilde{\mu} p + (n+1-c)\tilde{\xi} p'\right\} P_{n+1} = \left\{\tilde{\lambda}(1+\eta) + c\tilde{\mu} p + (n-c)\tilde{\xi} p'\right\} P_n - \tilde{\lambda}(1+\eta) P_{n-1}; \quad c \le n \le N-1 \qquad \text{... (3)}$$

$$\left\{c\tilde{\mu} p + (N-c)\tilde{\xi} p'\right\} P_N = \tilde{\lambda}(1+\eta) P_{N-1} \quad ; n = N \qquad \text{... (4)}$$

Solving equations (1) to (4), we obtain:

$P_n =$ Pr{n customers in the system}

$$P_n = \begin{cases} \dfrac{1}{n!}\left(\dfrac{\tilde{\lambda}(1+\eta)}{\tilde{\mu} p}\right)^n P_0 & ,1 \le n \le c \\ \dfrac{1}{c!}\left(\dfrac{\tilde{\lambda}(1+\eta)}{\tilde{\mu} p}\right)^c \left[\prod_{i=c+1}^{n}\left(\dfrac{\tilde{\lambda}(1+\eta)}{c\tilde{\mu} p + (i-c)\tilde{\xi} p'}\right)\right] P_0 & ,c \le n \le N-1 \end{cases} \qquad \text{... (5)}$$

$P_N =$ Pr{system is full}

$$= \frac{1}{c!}\left(\frac{\tilde{\lambda}(1+\eta)}{\tilde{\mu} p}\right)^c \left[\prod_{n=c+1}^{N}\left(\frac{\tilde{\lambda}(1+\eta)}{c\tilde{\mu} p + (n-c)\tilde{\xi} p'}\right)\right] P_0 \qquad \text{... (6)}$$

By using normalization condition, $\sum_{n=0}^{N} P_n = 1$

$P_0 =$ Pr{system is empty}

$$= \left[\sum_{n=0}^{c} \frac{1}{n!}\left(\frac{\tilde{\lambda}(1+\eta)}{\tilde{\mu} p}\right)^n + \sum_{n=c+1}^{N}\left\{\frac{1}{c!}\left(\frac{\tilde{\lambda}(1+\eta)}{\tilde{\mu} p}\right)^c \prod_{i=c+1}^{n}\left(\frac{\tilde{\lambda}(1+\eta)}{c\tilde{\mu} p + (i-c)\tilde{\xi} p'}\right)\right\}\right]^{-1} \qquad \text{... (7)}$$

3.3. Performance Measures

(i) Mean number of customers in the system (L_S)

$$L_S = \sum_{n=1}^{N} n\, P_n$$

(ii) Mean queue Length $\left(L_q\right)$

$$L_q = \sum_{n=c}^{N}(n-c)P_n$$

(iii) Waiting time in System $\left(W_S\right)$

$$W_S = \frac{L_S}{\tilde{\lambda}(1+\eta)}$$

(iv) waiting time in Queue (W_q)

$$W_q = \frac{L_q}{\tilde{\lambda}(1+\eta)}$$

(v) Average rate of Reneging $\left(R_r\right)$

$$R_r = \sum_{n=c}^{N}(n-c)\tilde{\xi}\, p' P_n$$

(vi) Average rate of Retention $\left(R_R\right)$

$$R_R = \sum_{n=c}^{N}(n-c)\tilde{\xi}\, q' P_n$$

4. Wingspans Fuzzy Ranking Method

Wingspans method is ranking fuzzy numbers depending on the region that lies

between the slope of the membership function and the horizontal real axis. Let $\phi_{\tilde{C}}$ be a membership function and c_0 be a core point of a fuzzy number $\tilde{c}$, Wingspans center of $\tilde{c}$ is

$$W_c = c_0 - \frac{1}{2}\int_{-\infty}^{c_0}\phi_{\tilde{C}}(x)\,dx + \frac{1}{2}\int_{c_0}^{\infty}\phi_{\tilde{C}}(x)\,dx$$

Let $\tilde{c} = \left[c_l, c_0, c_r\right]$ be a triangular fuzzy number. Wingspans center of $\tilde{c}$ and also our suggested fuzzy ranking function of triangular fuzzy number is

$$R(\tilde{c}) = \frac{1}{2}\left[c_0\right] + \frac{1}{4}\left[c_l + c_r\right]$$

4.1. Remark

(i) For any number in fuzzy, its Wingspans center is in existence and it is distinctive.

(ii) For any symmetric number in fuzzy, Wingspans center is just its symmetric center.

5. Numerical Example

Here our consideration is encouraged arrivals, revoking and retaining back out customers of a multi-server fuzzy feedback queuing model. The arrival rate $\tilde{\lambda}(1+\eta)$, service rate $\tilde{\mu}$, with reneging time distribution parameters $\tilde{\xi}$ are all fuzzy nature.

The number of service channels will be 3. We take the probabilities of reneging customer p' and retaining back out customer q' as follows: p' = 0, 0.1, …0.9, 1; q' = 1,0.9, …0.1, 0 with encouraging factor η = 0.5 and N = 10.

In the event of feedback being provided, post the accomplishment of the service, each customer who tends to re-join the system as a feedback customer to regularly undertake another service with probability p may not join with the probability $q = 1 - p$ .We take $p = 0.8$ and $q = 1 - p = 0.2$.

Ranking of Triangular fuzzy Numbers

Let us contemplate the arrival rate, service rate and time distribution parameters are triangular fuzzy numbers. Let $\tilde{\lambda}$ $=[4, 5, 6]$; $\tilde{\mu}$ = $[7, 8, 9]$ and $\tilde{\xi}$ = $[0.1, 0.2, 0.3]$ per hour respectively. The membership function of $\tilde{\lambda}$ = $[4, 5, 6]$ is defined by

$$\phi_{\tilde{C}}(x) = \begin{cases} \dfrac{(x-4)}{(5-4)} & , 4 \le x \le 5 \\ 1 & , x = 5 \\ \dfrac{(x-6)}{(5-6)} & , 5 \le x \le 6 \\ 0 & , otherwise \end{cases}$$

Thereby, we write the membership functions of $\tilde{\mu}$ and $\tilde{\xi}$ in the similar fashion. Subsequently, we apply Wingspans ranking for the triangular fuzzy numbers $\tilde{\lambda}$, $\tilde{\mu}$ and $\tilde{\xi}$.

R ($\tilde{\lambda}$) = R (4, 5, 6) = $\frac{1}{2}(5) + \frac{1}{4}(4+6)$ = 5

R ($\tilde{\mu}$) = R (7, 8, 9) = $\frac{1}{2}(8) + \frac{1}{4}(7+9)$ = 8

R ($\tilde{\xi}$) = R (0.1, 0.2, 0.3) = $\frac{1}{2}(0.2) + \frac{1}{4}(0.1+0.3)$ = 0.2

Recourse to this fuzzy ranking, we calculate the performance measures of described Queuing models and tabulate the values with respect to q'.

Table 1. Performance Measures Vs q'

S. No.	q'	p'	L_s	L_q	W_s	W_q	R_r	R_R
1	0	1	1.250511	0.081278	0.166735	0.010837	0.016256	0
2	0.1	0.9	1.251098	0.081623	0.166813	0.010883	0.014692	0.001632
3	0.2	0.8	1.251691	0.081971	0.166892	0.010929	0.013115	0.003279
4	0.3	0.7	1.252289	0.082322	0.166972	0.010976	0.011525	0.004939
5	0.4	0.6	1.252893	0.082680	0.167052	0.011024	0.009922	0.006614
6	0.5	0.5	1.253505	0.083042	0.167134	0.011072	0.008304	0.008304
7	0.6	0.4	1.254122	0.083407	0.167216	0.011121	0.006673	0.010009
8	0.7	0.3	1.254745	0.083777	0.167299	0.011170	0.005027	0.011729

9	0.8	0.2	1.255378	0.084152	0.167384	0.011220	0.003366	0.013464
10	0.9	0.1	1.256014	0.084531	0.167469	0.011271	0.001691	0.015216
11	1	0	1.256662	0.084917	0.167555	0.011322	0	0.016983

6. Result

With respect to the data in the above table, it is evident that the probability of retention q' increases, and likewise the mean number of customers in the system, looked-for customers in the queue, system's average waiting time, queue's average waiting time, as well as retention's average rate also increases considerably. But the average rate of reneging decreases. The customer retention $R_R = 0$ when $q' = 0.$ If q' is one, then the customer revoking R_r is zero. It implies that all reneging customers are retained in the queuing system.

Conclusion

This research proposed Wingspans method for ranking triangular fuzzy numbers pertaining to the region that lies between the slope of the membership function and the horizontal real axis. It ranks fuzzy numbers as possible and concrete. The performance measures of encouraged arrivals, revoking and retaining back out customers in a multi-server fuzzy feedback queuing model can be obtained by wingspans ranking method very effectively. This method not only provides crisp values but also accurate values as compared to other methods.

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