Original Article

FUZZY Β-SETS AND FUZZY ΒS –SETS IN FUZZY TOPOLOGICAL SPACES

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Abstract

Some new kinds of fuzzy subsets are defined such as the definition of fuzzy β- sets and fuzzy βs- set. Moreover, some properties, theorems and certain relations between them are proved., some interesting properties of fuzzy β- compactness are investigated. The purpose of this paper is devoted to introduce and study the concepts of β-compactness and β-closed spaces in fuzzy setting., A fuzzy set is one to which objects can belong to different degrees, called grades of membership. The fuzz y set as a number ranging from zero (absolutely false) to 1 (absolutely true). Before dealing with fuzzy systems preliminary definitions should be known. In these sections we will illustrate some important definitions and concepts.

Keywords: Fuzzy, βs- Set, Properties, Andrjvic, Topology.

INTRODUCTION

In many studies introduced the definitions of fuzzy sets and proved some their operations in his research for the first time in 1965. Andrjvic [1-4] established the definition of b-open sets in general topological space in 1969. Furthermore, in general topology and fuzzy topology, there are many researchers studied [5-9] different kinds of generalizations of continuous functions. Chang [10-13] defined the concept of fuzzy topological space in 1968. In this research, Section 3 we introduce the class of fuzzy β - sets and many studies [14-19] of their properties [20-22]. Additionally, in Section four we define the notion of fuzzy (βs- set and prove some theorems [23-25] and their many propositions [26-28].

SECTIONS

Preliminaries

In this section, we illustrate some basic definition and results.

Definition 2.1.[1] A family $\mathfrak{T}$ of fuzzy sets in μ is called fuzzy topology if it satisfies the following axioms:

(i) $0,1 \in \mathfrak{T}$
(ii) $\mu_1, \mu_2 \in \mathfrak{T}$, then $\mu_1 \wedge \mu_2 \in \mathfrak{T}$;
(iii) If $\{\mu_j : j \in J\} \subset \mathfrak{T}$, where J denotes an index set, then $\vee \mu_j \in \mathfrak{T}$.

The pair $(\mu, \mathfrak{T})$ is named as a fuzzy topological space or shortly f. t.s. The members of $\mathfrak{T}$ are defined as $\mathfrak{T}$ - open fuzzy set. If the complement of $\wp$ which denoted by $\wp^c$ is $\mathfrak{T}$ -open then an element $\wp \in [0,1]^X$ is said to be closed fuzzy set.

Definition 2.2. [9] A fuzzy set F in a fuzzy topological space (μ, 𝔗) is said to be fuzzy b-open set if F ≤ (int(cl(F)) ∨ (cl(int(F)) and we can denote by fb-open

Definition 2.3.[9] A fuzzy set F in a fuzzy topological space (μ, 𝔗) is said to be fuzzy b-closed set if F ≥ (int(cl(F)) ∧ (cl(int(F)))

Theorem 2.4.[9] For a fuzzy set F in a fuzzy topological space (μ, 𝔗) if F is fuzzy b -open set if and only if 1-F Is a fuzzy b-closed set.

Theorem 2.5. [6] For fuzzy set F in a fuzzy topological space (μ, 𝔗), F is fuzzy b-closed set if and only if 1-F is a fuzzy b-open set.

Definition 2.6.[6] Let F be fuzzy set in a fuzzy topological space (μ, 𝔗). Then its fuzzy b-closure if defined by Fbcl(F)= ∧{L ≥ F: L is a fb-closed set of (μ, 𝔗).

Definition 2.7.[6] let F be fuzzy set in fuzzy topological space (μ, 𝔗). Then its fuzzy b-interior if defined by Fbint(F)= ∨{G ≤ F: G is a fb-open set of (μ, 𝔗)}.

Theorem 2.8.[6] let F be any fuzzy set in a fuzzy topological space (μ, 𝔗) Then:

1) Fbcl(1-F) =1-fb int(F)
2) Fbint(1-F) =1-fbcl(F)

Remarks 2.9.[6]

1) if F is a fuzzy subset of fuzzy topological space (μ, 𝔗). Then Fbcl(F) is the smallest fb-closed set containing F. Thus Fbcl(F) = F ∨ (int(cl(F))) ∧ (cl(int(F)))
2) if F is a fuzzy subset of fuzzy topological space (L, 𝔗). Then Fbint(F) is largest fb-open set containing F. Thus Fbint(F)=F ∧ ((int (cl(F))) ∨ (cl(int(F)))

Theorem 2.10.[9] In a fuzzy topological space (μ, 𝔗)

1) An arbitrary union of fb-open set is a fb-open set.
2) An arbitrary intersection of fb-closed set is a fb-closed set.

Theorem 2.11.[6] In a fuzzy topological space (μ, 𝔗) the following relations hold

1) Fbcl(F ∨ U) ≥ Fbcl(F)∨Fbcl(U)
2) Fbcl(F∧U) ≤ Fbcl(F) ∧ Fbcl(U)

Definition 2.12.[7] A fuzzy subset A of a fuzzy topological space (μ, 𝔗) is said to be fuzzy Locally closed set if A=U∧V, where U∈ 𝔗and V is a fuzzy closed. The family of all fuzzy locally closed set is denoted by FLC(μ).

Definition 2.13.[7] The fuzzy topology $\mathcal{T}$ = {$0_μ$, $1_μ$, $\mathcal{M}, N$} on μ. Then $\mathcal{W} = \mathcal{M} \wedge \mathcal{N}$ is a fuzzy locally closed set, but it can't be fuzzy open.

Proposition 2.15.[7] An intersection of a fuzzy locally closed set with any fuzzy open set is fuzzy locally closed.

Proposition 2.16.[7] If $\mathcal{W}$ *and* $\mathcal{V}$ *ar*e fuzzy locally closed subsets of f. t.s. (μ, 𝔗), therefore $\mathcal{W} \wedge \mathcal{V}$ is fuzzy locally closed.

Proposition 2.17.[7] A fuzzy set $\mathcal{W}$ in any f. t.s. (μ, 𝔗) is fuzzy locally closed if and only if $\mathcal{W} = \mathcal{M} \wedge \mathcal{W}$ for some fuzzy open set $\mathcal{M}$

Definition 2.18. A space μ is called fuzzy b – closed space if every fb – open subsets of f. t.s. (μ, 𝔗) is fuzzy open sets.

Definition 2.18. A f. t.s. (μ, 𝔗) is said to be fuzzy extremally disconnected if the fuzzy closure of every fuzzy open subset of μ is fuzzy open.

Fuzzy β-sets

Definition 3.1. A subset A of a fuzzy topological space (μ, $\mathfrak{T}$) is said to be fuzzy β-set if A is fuzzy b-open and fuzzy locally closed set. The family of all β - sets is denoted by $F\beta_s(\mu)$. It is clear that $F\beta_s(\mu)$ = FLC (μ, $\mathfrak{T}$) ∧ FBO (μ, $\mathfrak{T}$).

Proposition 3.2. Every open subset of topological space (μ, $\mathfrak{T}$) is Fβ-set.

The proof is obvious.

Remark 3.3. The converse of Proposition (3.2) is not true in general.

Proposition 3.4. If μ is a submaximal space, then the union of any family of fuzzy β-sets is fuzzy β-set.

proof: let $\{A_\alpha\}$, $\alpha \in \Omega$ be a family of all fuzzy β - sets in f. t.s. (μ, $\mathfrak{T}$). Since A_α is fuzzy β-set, $\forall \alpha \in \Omega$, so is fuzzy b - open, $\forall \alpha \in \Omega$. Therefore, $\{A_\alpha\}$, $\alpha \in \Omega$ is union of fb - open sets.

Hence, from Theorem (2.10), we get, $\vee A_\alpha$ is fuzzy b - open and since μ is submaximal space $\vee A_\alpha$, $\alpha \in \Omega$ is closed set. Therefore, $\{A_\alpha\}, \alpha \in \Omega$ is fuzzy β – set.

Proposition 3.5. The intersection of a fuzzy open set and fuzzy β-set is fuzzy β-set.

Proof: Let N be a fuzzy open set and M is fuzzy β - set in f. t.s. (μ, $\mathfrak{T}$). Then by theorem (2.10), we get N ∧ M is fuzzy b – open. Since M is fuzzy locally closed so there exist $U \in \mathfrak{T}$ and L is fuzzy closed subset in μ such that M = U ∧ L since N ∧ M = N ∧ (U ∧ L) = (M ∧ U) ∧ L is fuzzy locally closed (since N ∧ U is fuzzy open). Hence, N ∧ U is fuzzy β - set.

Remark 3.6. The intersection of two fuzzy β - set need not to be fuzzy β - set in general.

Theorem 3.7. Let (μ, $\mathfrak{T}$) be extremally disconnected fuzzy topological space. Then A ≤ μ is fuzzy open If and only if A is fuzzy β - set.

Proof: The necessity proof is obvious.

To proof sufficiency, let A be fuzzy β-set then, A is fuzzy b-open and locally closed, so

A ≤ int(cl(A)) ∨ cl(int(A)) and A= U ∧ cl(A) where, U∈ $\mathfrak{T}$.

A ≤ U ∧ (int(cl(A)) ∨ cl(int(A))

≤ [int (U ∧ cl(A))] ∨ [U ∧ cl(int(A))]

= [int (U ∧ cl(A))] ∨ [U ∧ int(A))]

≤int(A) ∨ [U ∧ int(cl(A))]

≤ int(A) ∨ int [U ∧ cl(A))]

= int(A) ∨ int(A) = int(A)

Therefore, A is open

Fuzzy βs - sets

Definition 4.1. A subset A of a f. t.s. (μ, $\mathfrak{T}$) is called a fuzzy β_s - set if A ∧ B ∈ $F\beta_s(\mu)$ for all B ∈ $F\beta_s(\mu)$ The class of all $F\beta_s$ - set in (μ, $\mathfrak{T}$) will be denoted by $\mathfrak{T}_{F\beta s}$.

Remark 4.2. we will denote to the complement of $F\beta_s$ - set be 1- $F\beta_s$ – set.

Theorem 4.3. If μ is submaximal space, then $\mathfrak{T}_{F\beta s}$ is a topology on μ.

Proof:

(1) Clearly $\emptyset$ and $\mu \in \mathfrak{T}_{F\beta s}$,

(2) Let $\{A_\alpha \mid \alpha \in \Omega\} \subseteq \mathfrak{T}_{F\beta s}$. Then, $A_\alpha \wedge B \in F\beta_s(\mu)$ for all $B \in F\beta_s(\mu)$ and for each $\alpha \in \Omega$. Therefore, $(\vee_{\alpha \in \Omega} A_\alpha) \wedge B = \vee_{\alpha \in \Omega}\{A_\alpha \wedge B\}$ where $A_\alpha \wedge B$ is fuzzy β_s-set and since arbitrary union of Fβ-set, If follows that $\vee_{\alpha \in \Omega}\{A_\alpha \wedge B\}$ is Fβ-set for each $B \in F\beta_s(\mu)$ and hence $(\vee_{\alpha \in \Omega} A_\alpha) \wedge B$ is Fβs-set. Therefore, $\vee_{\alpha \in \Omega} A_\alpha \in \mathfrak{T}_{F\beta s}$,

(3) Let K, L $\in \mathfrak{T}_{F\beta s}$ Then, $(K \wedge L) \wedge G = K \wedge (L \wedge G)$ for all $G \in F\beta_s(\mu)$, hence $K \wedge L \in \mathfrak{T}_{F\beta s}$. Therefore, form (1), (2) and we have $\mathfrak{T}_{F\beta s}$ is topology on μ

Theorem 4.4. If μ is a submaximal space, then $\mathfrak{T}_{F\beta s} \subseteq F\beta_s(\mu)$.

Proof

Let $G \in \mathfrak{T}_{F\beta s}$, then $G \wedge L \in F\beta_s(\mu)$ for all $B \in F\beta_s(\mu)$. particularly, if we put B = μ, then we get $F \wedge \mu = F$ is β - set and $F \in F\beta_s(\mu)$. Hence, $\mathfrak{T}_{F\beta s} \subseteq F\beta_s(\mu)$.

Theorem 4.5. If μ is a submaximal space, then $\mathfrak{T} \subseteq \mathfrak{T}_{F\beta s}$.

proof:

let (μ, $\mathfrak{T}$) be topological space and consider $L \in \mathfrak{T}$. Then, from Proposition (3.5) for any fuzzy β-set G in μ we have $L \wedge G$ is fuzzy β - set. Therefore, L is $F\beta_s$-set, so $L \in \mathfrak{T}_{F\beta s}$. Hence, $\mathfrak{T} \subseteq \mathfrak{T}_{F\beta s}$.

Theorem 4.6. If topological space μ is a submaximal extremally disconnected, then $\mathfrak{T}_{F\beta s} = F\beta_s(\mu) = \mathfrak{T}$.

proof:

It has been found from Theorem (4.4) that a $\mathfrak{T}_{F\beta s} \subseteq F\beta_s(\mu)$. Now, we went to prove $F\beta_s(\mu) \subseteq \mathfrak{T}_{F\beta s}$. Let $G \in F\beta_s(\mu)$, then G is fuzzy open set (Since μ is a submaximal extremally disconnected). Hence, by Theorem 4.5. $\mathfrak{T} \subseteq \mathfrak{T}_{F\beta s}$, so $G \in \mathfrak{T}_{F\beta s}$. Therefore, $F\beta_s(\mu) \subseteq \mathfrak{T}_{F\beta s}$. Therefore, we get $\mathfrak{T}_{F\beta s} = F\beta_s(\mu) = \mathfrak{T}$.

Theorem 4.7. Let (μ, $\mathfrak{T}$) is a submaximal Fb-space. Then, $\mathfrak{T}_{F\beta s} = \mathfrak{T}$.

proof:

Let (μ, $\mathfrak{T}$) is fuzzy b-space and consider $L \in \mathfrak{T}_{F\beta s}$. But, from Theorem 4.4 we get,

$\mathfrak{T}_{F\beta s} \subseteq F\beta_s(\mu)$. Hence, L is a β-set. But μ is Fb-space. So that, L is fuzzy open set in μ and so $L \in \mathfrak{T}$. Hence, $\mathfrak{T}_{F\beta s} \subseteq \mathfrak{T}$ and since $\mathfrak{T} \subseteq \mathfrak{T}_{F\beta s}$. Therefore, $\mathfrak{T}_{F\beta s} = \mathfrak{T}$.

CONCLUSION

In this research, we discussed in Sections we introduce the class of fuzzy β - sets and study some of their properties. Additionally, in Section 4 we define the notion of fuzzy βs- set and prove some theorems and propositions.

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Authors Contributions

The author carried out and contributed in all writing, analysis.

Conflicts of Interests

The authors declare that there is no conflict of interest in this research

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