Original Article

# PRE-BUCKLING STRUCTURAL ANALYSIS OF THIN Z-SECTION UNDER UPLIFT LOADING

**Dr. Amit S. Chaudhary[1], Dr. Kishor. B. Waghulde[2], Dr. Vijaykumar Javanjal[3], Dr. Subhash Gadhave[4], Dr. Kashinath Haribhau Munde[5], Dr. Atul Ashok Patil[6]**

[1]Associate. Professor, Department of Mechanical Engineering, DIT, Pimpri, Pune, India.
[2]Professor, Department of Mechanical Engineering, DIT, Pimpri, Pune, India.
[3]Associate. Professor, Department of Mechanical Engineering, DIT, Pimpri, Pune, India.
[4]Associate. Professor, Department of Mechanical Engineering, DIT, Pimpri, Pune, India.
[5]Professor, Department of Mechanical Engineering, APCOER, Parvati, Pune, India.
[6]Professor, Department of Mechanical Engineering, DIT, Pimpri, Pune, India.

**DOI:** 10.47750/pnr.2023.14.02.346

## Abstract

A z-section purlin steel sheet is considered in this paper to understand its structural behavioragainst the wind uplift loading. Generally toUDL is considered for a practical & effectivedesign as main aspect of loading. A verylimited research is done on combined bendingand lateral torsional buckling, so it is importantto investigate the impact of combine loading onthe z-section which is partially restrained andapplied with uplift loading. In this to describelateral torsional behavior and bending, ananalytical model is developed for the z-sectionwhich is partially restrained. Thin walledclassical bending theory is used to derive thebending stress formula for the z-section. Thepresent developed analytical model thencompare with the Eurocode EN1993-1-3 modeland with FEA results.

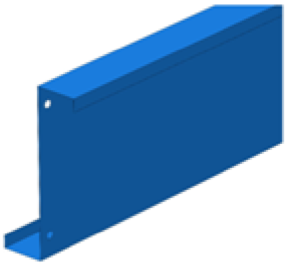
**Keywords:** Cold-formed steel Z-sections; thinwalledstructures, pre-buckling, lateral torsional buckling, finite element method,analytical method, uplift load.

## 1.0 Introduction

Generally for structure building z-section steelcold formed material is used. Sheeting, Roofsections or purlins, and shade are theextensively used in structural building. Thementioned things are not used directly or asmajor part of the structure but can be used asintermediate parts or members in between themain structure. Z-sections, C-sections, Isectionsare most commonly used and toimprove efficiency, provided with theadditional element called as lips. It is a lightweight, large strength compare to weight,flexible, easy to produce and shape and mostimportantly can produce in variety in shapes and this is the reason recently they are verypopular in structural building with wastereduction and cost effective design.

However hot rolled and cold rolled sections arecannot be comparable as cold rolled is thinwalled component whether hot rolled is thickand care must be taken while designing withcold roll. Cold roll have its own disadvantageslike its uses is reduced at the time of complexmechanical design. Slight cold-shaped steelindividuals start as thick, liquid, hot steelpieces. Every chunk is ordinarily hot-moved,cold-decreased, and toughened before loopingand transporting the dainty steel sheet to rollshapingmakers (Moen et al., 2008). Theprimary deliberate and completely recordedinvestigation into the conduct of cold-shapedsteel areas was done by Winter at CornellUniversity during the time of the Second WorldWar (Winter, 1947; 1949). This brought aboutthe principal plan particular managing coldframedsteel segments in 1946 (AISI, 1946). Inthe UK, investigation into this field started inthe mid 1950s (for instance, Chilver, 1953a;1953b; Kenedi and Harvey, 1951; Harvey,1952; 1953). The principal British Specificationwas distributed in 1959 (BSI, 1959), in spite ofthe fact that the proposition for the structureparticular had been made eight years sooner(Shearer, 1951). Clasping of auxiliaryindividuals is a significant part of strongmechanics. The least difficult models are theclasping of the swagger, concentrated first byEuler in 1757 (Timoshenko, 1953), and theclasping of the level plate concentrated byNavier in 1823 (Timoshenko, 1953) and laterby Bryan in 1891 (Timoshenko, 1953). It wasmaybe Lundquist and Stowell (1943) whooriginally recognized the distortional-likeclasping mode when they examined theclasping of a limited level plate with a durablestiffener along an edge of the plate. Afterward,Gallaher and Boughan (1947) likewise found acomparative distortional locking mode in zedhardenedboards. Distortional locking in coolframed steel segments was first found by Vander Maas (1954) and detailed the event of"another sort" of neighborhood claspingwonder including rib stiffener turns. In thefollowing couple of decades, scientist(Dwight,1963, Sharp, 1966) kept on buildingup certain techniques to comprehend thisclasping. In the late 1970s and mid 1980s,Desmond (1977, 1981a and 1981b) introduceda few discoveries on the distortional claspingconduct of lipped channel bars and segments.Simultaneously, Thomasson (1978) additionallygave outcomes to uncover the presence ofdistortional locking in lipped channel segments.

Figure 1 Cleat bolted supported boundary condition of zed- and channel-sections.



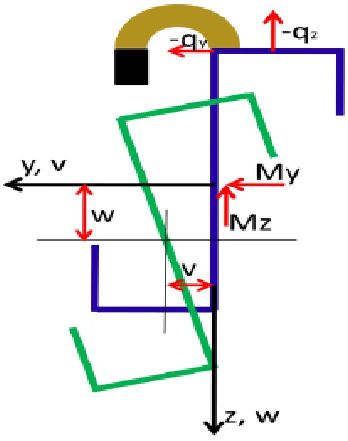
## 2.0 FEA Pre-buckling Analysis

### 2.1 Pre-buckling stress analysis of z sections

### 2.1.1 Analytical model and calculation

A purlin section is considered with upper flange is connected with sheeting with self tapping fastener of the section and bottom flange is remains free. The attached sheeting is act as restraints to lateral and rotational movement.

Figure 2 Analytical model used for a z section Purlin-sheeting system.



Let x,y,z be the origin (coordinate system) and consider that as the shear center of the section in cross. X-axis is considered as beam longitudinal axis. The equilibrium equation from the bending theory (Timoshenko and Gere, 1961) expressed in the form of displacement is mentioned below,

$$EI_z\frac{d^4v}{dx^4}+EI_{yz}\frac{d^4w}{dx^4}=q_y \qquad (1)$$

$$EI_{yz}\frac{d^4v}{dx^4}+EI_y\frac{d^4w}{dx^4}=q_z \qquad (2)$$

$$EI_w\frac{d^4\emptyset}{dx^4}-GI_T\frac{d^2\emptyset}{dx^2}+k_\emptyset\emptyset=z_kq_y-aq_z \qquad (3)$$

For long beam the valid governing equitation's are (1) to (3) coz of the consideration that even after deformation the cross section of the beam remains same, but for short beam the equation is not suitable. By using equations (1) and (2) to eliminate qy and w from equation (3), it yields.

$$EI_w\frac{d^4\emptyset}{dx^4}-GI_T\frac{d^2\emptyset}{dx^2}+k_\emptyset\emptyset-z_k\left[EI_z\frac{d^4v}{dx^4}+\frac{I_{yz}}{I_y}\left(q_z-EI_{yz}\frac{d^4v}{dx^4}\right)\right]=-aq_z \qquad (4)$$

The restraint for lateral displacement is subjected to top flange only

$zk\emptyset + v = 0$ (5)

By eq. (3) to (5) to eliminate the twist angle, f, in eq.(3) to (4), it yields,

$$\left(EI_Z-\frac{EI_{yz}^2}{I_y}+\frac{EI_w}{Z_k^2}\right)\frac{d^4v}{dx^4}-\frac{GI_T}{z_k^2}\frac{d^2v}{dx^2}+\frac{k_\emptyset}{z_k^2}v$$

$$=q_z\left(\frac{a}{z_k}-\frac{I_{yz}}{I_y}\right) \qquad (6)$$

Let,

$$I_{eq}=\frac{1}{4}\left(I_z-\frac{I_{yz}^2}{I_y}+\frac{I_w}{z_k^2}\right) \qquad (7)$$

$$k_h=\frac{1}{2}\left(\frac{I_{yz}}{I_y}-\frac{a}{z_k}\right) \qquad (8)$$

For Z section of equal flanges $zk$ = h/2, where h is the depth, the equation (6) can be written as,

$$EI_{eq}\frac{d^4v}{dx^4}-\frac{GI_T}{h^2}\frac{d^2v}{dx^2}+\frac{k_\emptyset}{h^2}v=-\frac{k_hq_z}{2} \qquad (9)$$

For simply supported beam, the solution for eq. (9) in x and y axis direction is expressed by (approximately)

$$v(x)=\sum C_m sin\frac{m\pi x}{l} \qquad (10)$$

Where, Cm is the constant to be determined and *l* is the beam length. By putting eq. (10) in eq. (9), we get

$$C_m = \frac{\frac{k_h q_z}{m\pi}\left(1-(-1)^m\right)}{EI_{eq}\left(\frac{m\pi}{l}\right)^4 + \left(\frac{GI_T}{h^2}\right)\left(\frac{m\pi}{l}\right)^2 + \frac{k_\emptyset}{h^2}} \quad (11)$$

Cross sectional horizontal component of the displacement at the shear center coz of UDL can be expressed as,

$$v(x) = \sum_{m-1,3,5} \frac{2k_h q_z l^4}{m\pi^5 EI_{eq}} * b \qquad (12)$$

$$b = \frac{1}{m^4 + \frac{GI_T}{EI_{eq}}\left(\frac{ml}{\pi h}\right)^2 + \frac{K_\emptyset l^4}{\pi^4 EI_{eq} h^2}} \frac{m\pi x}{l}$$

Because of displacement, the bending stress generated can be calculated as,

$$\sigma_x(x,y,z) = -Ey\frac{d^2v}{dx^2} - Ez\frac{d^2w}{dx^2} + E(\overline{\omega}-\omega)\frac{d^2\emptyset}{dx^2} \qquad (13)$$

Where ω = the sectorial coordinate with respect to the shear centre; $\overline{\omega}$ = the average value of ω.

Stress generated due to vertical deflection of beam is the first term in the right hand side of the eq (13), stress generated due to horizontal direction is second term, and warping stress is the third term. The component of displacement w is split into 2 parts (i.e w=wo+w1) and from eq (2) these 2 parts can be calculated as,

$$EI_y\frac{d^4w}{dx^4} = q_z \qquad (14)$$

$$EI_{yz}\frac{d^4v}{dx^4} + EI_y\frac{d^4w_1}{dx^4} = 0 \qquad (15)$$

With the use of eq. (5), eq. (14) and (15), Eq.(13) can be rewritten as,

$$\sigma_x(x,y,z) = -l - m \qquad (16)$$

$$l = Ez\frac{d^2w_0}{dx^2}$$

$$m = E\left(y - \frac{zI_{yz}}{I_y} + \frac{\overline{\omega}-\omega}{z_k}\right)\frac{d^2v}{dx^2}$$

The total bending stress can be decomposed in

2 parts as indicated in eq. (16)

$$\sigma_{x1}(x,y,z) = -Ez\frac{d^2w_0}{dx^2} \qquad (17)$$

To calculate the stress generated by the beam horizontal direction deflection caused by the load - $k_h q_z$

$$\sigma_{x2}(x, y, z) = -E\left(y - \frac{zI_{yz}}{I_y} + \frac{\bar{\omega}-\omega}{z_k}\right)\frac{d^2v}{dx^2} \quad (18)$$

The bending stress $\sigma_{x1}$ for a simply supported beam can be expressed as follows

$$\sigma_{x1}\left(\frac{l}{2}, y, z\right) = \frac{zM_{y,max}}{I_y} \qquad (19)$$

The bending stress $\sigma_{x2}$ is obtained by substituting Eq. (12) into (18), that is

$$\sigma_{x2}\left(\frac{l}{2}, y, z\right) = \frac{k_R k_h M_{y,max}}{I_{eq}}(n - o + p) \ (20)$$

$$n = \left(\frac{y}{2}\right), o = \left(\frac{zI_{yz}}{2I_y}\right), p = \left(\frac{\bar{\omega}-\omega}{h}\right)$$

where *kR* = the correction factor for the largest moment of the beam bent about the z-axis, considering the influence of the rotational spring and torsional rigidity of the section on the maximum moment, which is defined as follows,

$$k_R = \frac{32}{\pi^3}\sum_{m-1,3,5}(q) \qquad (21)$$

$$q = \frac{(-1)^{\frac{m-1}{2}} m}{m^4 + \frac{GI_T}{EI_{eq}}\left(\frac{ml}{\pi h}\right)^2 + \frac{K_\emptyset l^4}{\pi^4 EI_{eq} h^2}}$$

It is obvious from Eq. (21) that, if $k\varphi = 0$ and $I_T = 0$ then $kR = 1$, otherwise, if $k\varphi = \infty$ or $I_T = \infty$ then $kR=0$.

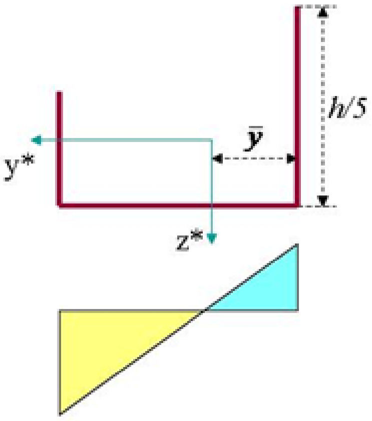
### *2.1.2 Comparison with EN1993-1-3*

The z-section purlin bending stress as per

Eurocode (EN1993-1-3, 1993), is calculatedwith the help of two different types as, one inwhich the beam is bent only in the plane of weband the other is in which beam bent about anaxis parallel to the z-axis, and the stress iscalculated as,

$$\sigma_{x2}\left(\frac{l}{2}, y, z\right) = \frac{k_R^* k_h M_{y,max}}{I_{fz}}(y - \bar{y}) \quad (22)$$

Figure 3 Bending stress calculated in EN19931-3.



For simply supported purlin, the moment correction factor as er Eurocode for anti-sag bar is given by,

$$k_R^* = \frac{1 - 0.0225R}{1 + 1.013R} \qquad (23)$$

$$R = \frac{Kl^4}{\pi^4 EI_{fz}} \qquad (24)$$

$$K = \left(\frac{4(1 - v^2)h^2(h + b_{mod})}{Et^3} + \frac{h^2}{k_\emptyset}\right)^{-1} \qquad (25)$$

Where, *bmod*=a*'* is for cases where the equivalent lateral force (-*khqz*) bringing the purlin into contact with the sheeting at the purlin web.

Table 1: List of differences between the present and EN1993-1-3 models.

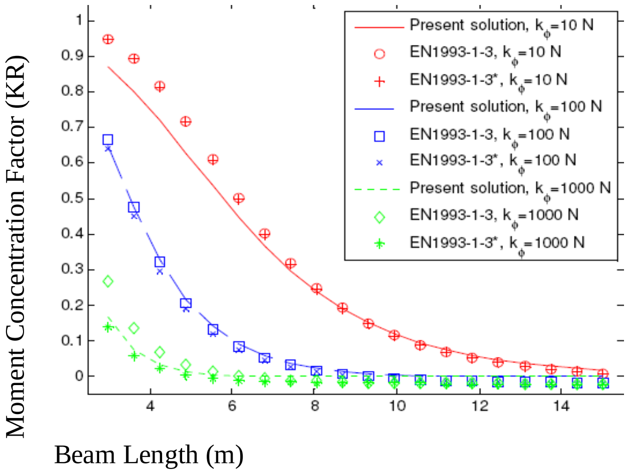
| Particulars | Present Model | EN1993-1-3 |
|---|---|---|
| Formula to calculate moment correction factor | Equation 21 | Equation 23 |
| Definition of the 2nd moment used to calculate bending stress | $I_{eq}$<br>Equation 7 | $I_{fz}$<br>Figure 3 |

| | | |
|---|---|---|
| Coordinate used to calculate bending stress | $\left(\frac{y}{2}-\frac{zI_{yz}}{2I_y}+\frac{\overline{\omega}-\omega}{h}\right)$ | $(y-\bar{y})$ |

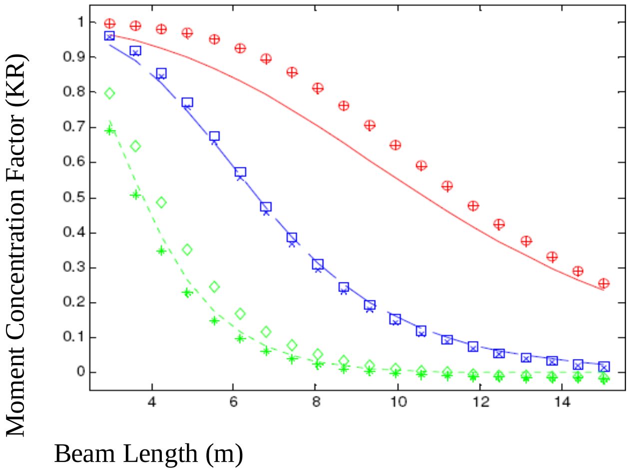
The comparison between moment correctionfactor (MCF) by calculating with EN1993-1-3 and with calculating present model is asfollows,

- MCF decreases due to increase ineither torsional spring stiffness orbeam length.
- The MCF decrease rate in smallsection is fast that in large section.
- The calculated value of MCF byEN1993-1-3 is more than presentmodel due to consideration oftorsional rigidity in present model.

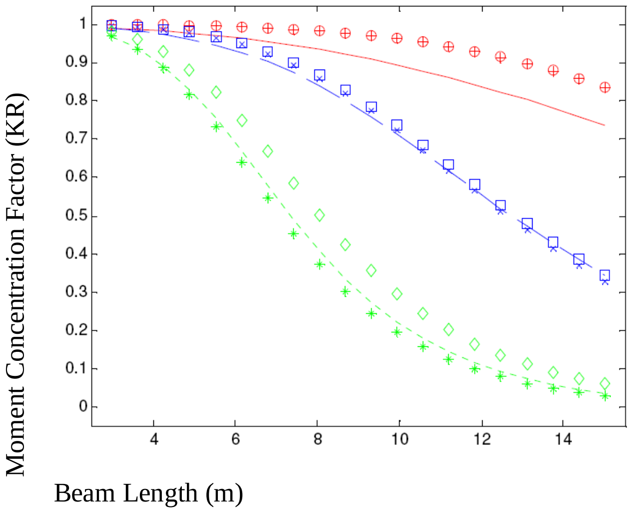
Figure 4 Comparison of moment correction factors between the present and EN1993-1-3 models.The bending stress σx2 can be express as follows,



(a) h=120mm, b=50mm, c=15mm, t=1.5mm, a=b/2.



(b) h=200 mm, b=75 mm, c=20 mm, t=2 mm, a=b/2.



(c) h=400mm, b=100mm, c=30mm, t=2.5mm, a=b/2.

$$\sigma_{x2}(x,y,z) = -E\frac{d^2 2v}{dx^2}(y-\bar{y}) \qquad (26)$$

Substituting equation 12 into equation 26, we get

$$\sigma_{x2}(x,y,z) = -\frac{k_R k_h M_{y,max}}{I_{eq}}(y-\bar{y}) \quad (27)$$

Figure 5 shows the comparison between the ratio of the equivalent second moment (ESM) of cross-section area with En1993-1-3 and in present models is as follows,

- ESM is small by EN1993-1-3 than by the present model.
- Bending stress by EN1993-1-3 is higher than by the present model.

The warping stress is does not take into account in EN1993-1-3 whereas taken in present model. Also comparison of bending stress due to lateral displacement and lateral load is shown in figure no 6.

Figure 5: Ratio of the equivalent second moments of cross-section area used in the present (*Ieq*) and the EN1993-1-3 (*Ifz*) models for 60 zed-sections.

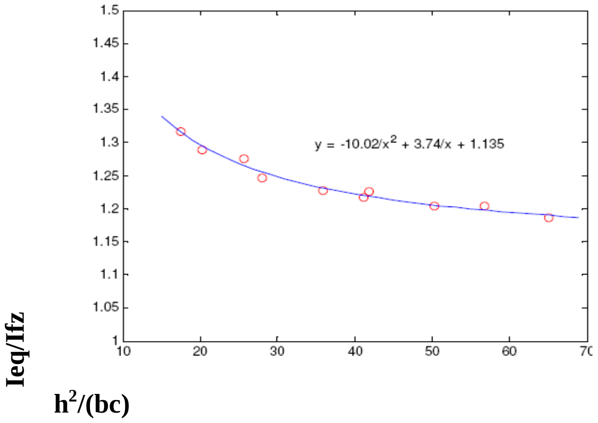
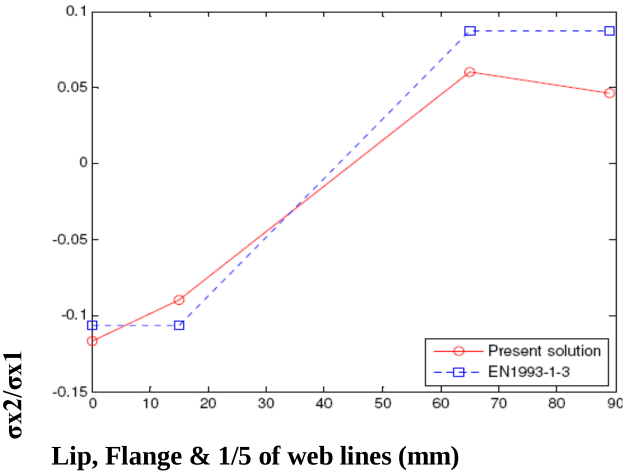
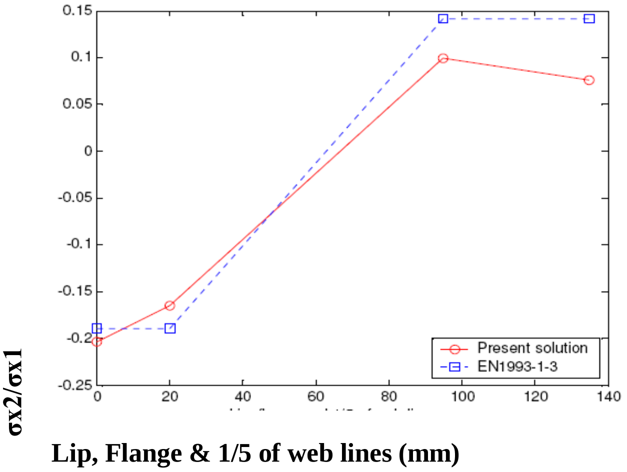


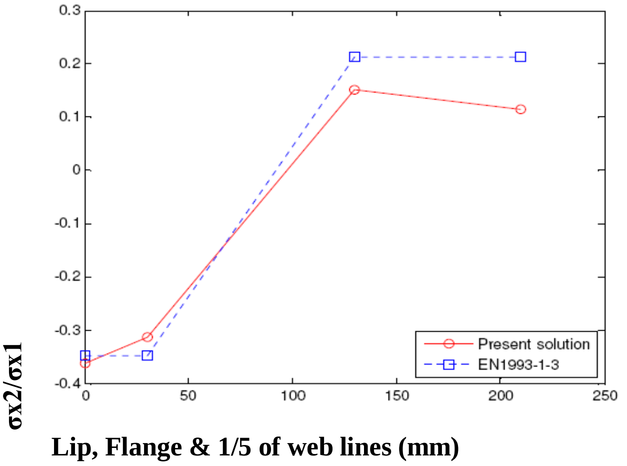
Figure 6: Bending stress distribution along thelip, flange and web lines (abscissa starts fromthe tip of lip and ends at 1/5 of the weblength).



(a) h=120 mm, b=50 mm, c=15 mm, t=1.5 mm, a=b/2.



(b) h=200 mm, b=75 mm, c=20 mm, t=2 mm, a=b/2.



(c) h=400 mm, b=100 mm, c=30 mm, t=2.5 mm, a=b/2.

## 3.0 Finite element model and analysis

To simulate the proposed model, FEA of a z section purlin beam of length *L*, web depth *h*, flange width *b*, lip length *c* and thickness *t* is tested for different values (see Figure 7).

Figure 7 Zed-section.

(a) Section dimensions (b) Uplift loading

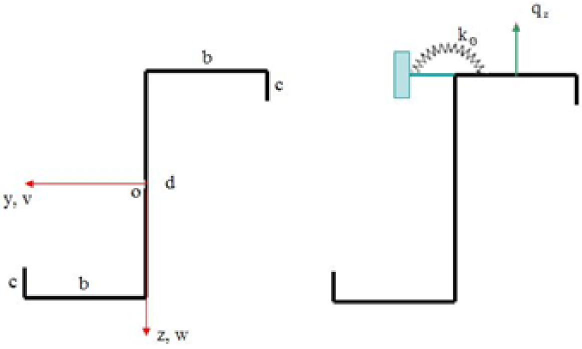


Figure 8 FE analysis of Z-section

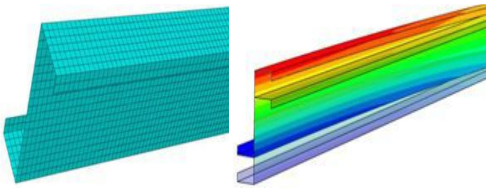
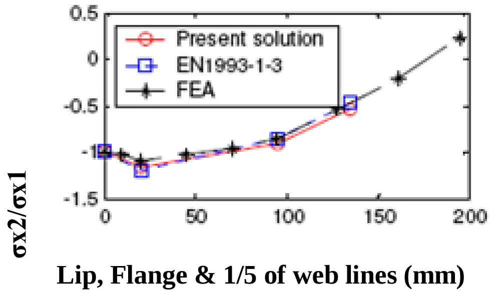
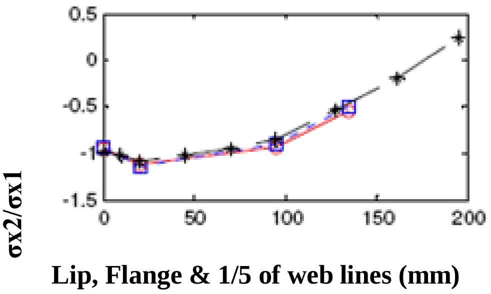


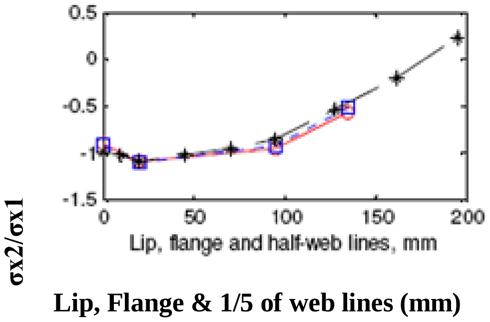
Figure 9 Bending stress distribution along the lip, flange and web lines (L=4000 mm)



(a) h=120mm, b=50mm, c=15mm, t=1.5mm, a=b/2.



(b) h=200mm, b=75mm, c=20mm, t=2mm, a=b/2.



(c) h=400mm, b=100mm, c=30mm, t=2.5mm, a=b/2.

## Conclusion

To describe the partially restrained cold formed steel purlins twisting and bending behavior an analytical model is presented in this paper. By using beam thin walled classical bending theory bending stress formula has been derived. The important concluding remarks from the study is summarized below,

- The values of bending stresses are close enough for both (present and EN1993-1-3) methods.
- There is a good agreement between web simply supported boundary conditions and cleats bolted supported. For pre-buckling analysis, in this present study idealized boundary conditions are proposed.
- Comparison shows that, although the bending stresses in the compression flange for the lateral bending are over-predicted by EN1993-1-3, the total bending stresses are still accurate enough, due to the bending stresses being dominated by the bending in the plane of the web rather than by lateral bending.
- For z-sections, the FEA outcomes have demonstrated that the total bending stresses predicted using the present and the EN1993-1-3 models are accurate for medium and long beams. Only for short beams with a high stiffness of rotational spring, are the bending stresses in lip and part of the flange over-predicted by the two analytical models.
- The longitudinal stress induced by lateral bending is significant for z section purlins. This additional stress may change the failure modes from lateral-torsional buckling to local or distortional buckling.

## REFERENCES

1. Barbero, E. J., Fu, S. H., and Raftoyiannis, I. "Ultimate bending strength of composite beams.", Journal of Materials in Civil Engineering Vol.3 No.4, 1991, pp. 292–306.
2. Drummond, J. A., and Chan, W. S., "Fabrication, Analysis, and Experimentation of a Practically
1. Constructed Laminated Composite I Beam under Pure Bending", Journal of Thermoplastic Composite Materials, May 1999, pp. 177-187.

2. K. B. Waghulde, A. A. Patil, and K. K.Dhande “Plastic part optimization for warpage using taguchi method”, AIP conference proceding. 2018.
3. Amit S. Chaudhary, "Structural analysis of a straight cantilever beam with openings in trapezoidal web profile for eccentric loading through FEA", International conference on computer science and mechanical engineering, 16 Aug 2016.
4. Craddock, J. N., and Yen, S. C., “The bending stiffness of laminated composite material Ibeams”, Composite Engineering Vol. 3 No.11, 1993, pp. 1025–1038.
5. Chandra, R., and Chopra, I., “Experimental and theoretical analysis of composites I-beams with elastic couplings”, AIAA Journal Vol. 29 No.12, 1991, pp. 2197–2206.
6. B. Kumar, K. B. Waghulde, “Vibration analysis of cantilever smart structure by using piezoelectric smart material” International journal on smart sensing and intelligent system. 2011.
7. Chaudhary A, "A parametric study on effect of trapezoidal web profile on the angle of twist of steel i-beam for eccentric loading", International journal on advanced in engineering and science, Volume no 3, Special issue no 01, Sept 2015.
8. A. S. Chaudhary, "Design & Analysis of Varying Cross Sectional Cantilever Beam with Trapezoidal Web for Jib Cranes" International Engineering Research Journal (IERJ) Special Issue 2 Page 4475-4480, 2015, ISSN 2395-1621
9. Barbero, E. J., Lopez-Anido, R., and Davalos, J. F., “On the mechanics of thin-walled laminated composite beams.”, Journal of Composite Materials Vol.27 No.8, 1993, pp. 806–829.
10. Song, O., Librescu, L., and Jeong, N. H., “Static response of thin-walled composite I-beams loaded at their free-end cross section: analytical solution”, Composite Structures Vol.52, 2001, pp. 55–65.
11. Jung, S. N., and Lee, J. Y., “Closed-form analysis of thin-walled composite I beams considering non-classical effects.”, Composite Structures Vol.60, 2003, pp. 9–17.
12. Chan, W. S., and Chou, C. J., “Effects of Delamination and Ply Fiber Waviness on Effective Axial and Bending Stiffnesses in Composite Laminates”, Composite Structures, Vol. 30, pp. 299-306.
13. Chan, W. S. and Demirhan, K. C., “A Simple Closed Form Solution of Bending Stiffness of Laminated Composite Tubes”, Journal of Reinforced Plastics and Composites, Vol.19 No. 3, 2001.
14. B. Sinha, M. M. Patil, K. B. Waghulde, “Vibration control of cantilever smart beam by using piezoelectric actuators and sensors”, International journal of engineering and technology, 2010.
15. Rojas, C. A., Syed, K. A., and Chan, W. S., “Structural Response of Composite Truss Beams”, The 22nd Annual Technical Conference of American Society of Composites, Sept. 2007.
16. Syed, K.A. and Chan, W.S., “Analysis of Hat-Sectioned Reinforced Composite Beams”, Proceedings of American Society of Composites, Sept. 2006. 18. Rios, G., “A Unified Analysis of Stiffener Reinforced Composite Beams with Arbitrary Cross-Section”, The University of Texas at Arlington, 2009.
17. Gan, L. H., Ye, L., and Mai, Y. W., “Simulations of mechanical performance of pultruded I beams with various flange web conjunctions”, Composites Part B Vol.30, 1999, pp. 423–429.
18. Daniel, I. M., and Ishai, O, Engineering Mechanics of Composite Materials, 2nd edition, Oxford University Press, New York, 2006.
19. M. Nurhaniza1 and M.K.A. Ariffin, "Finite element analysis of composites materials for aerospace applications", 9th National Symposium on Polymeric Materials (NSPM 2009), IOP Conf. Series: Materials Science and Engineering 11 (2010) 012010.