

On The Domination Variants Of Some Degree Based Topological Indices

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Abstract

Domination is an ancient graph theoretical topic with applications in psychology, political science, and human behaviour. A topological index is an area of graph theory that governs a compound's molecular structure. This work aims to conglomerate the two different branches and determine the domination variants of some celebrated degree based topological index. The newly described descriptors are generalised to standard graphs, corona products of graphs, and a few members of the windmill family of graphs in this study.

Keywords: Domination, molecular descriptor, Corona Product, Rheological property

INTRODUCTION

Many researchers are interested in topological indices because of their use in numerous fields of chemistry and biology. The branching index[1] was the first degree specified molecular descriptor, and many more thereafter[2],[3]. The degree biased descriptors used in the present study are mentioned below.

The Harmonic index[6] of a graph G is given by

$$HI(G) = \sum_{uv \in E(G)} \frac{2}{d_G(u) + d_G(v)}$$

The Geometric Arithmetic index[5] of a graph is defined as

$$GA(G) = \sum_{uv \in E(G)} \frac{\sqrt{d_G(u)d_G(v)}}{\left[\frac{d_G(u) + d_G(v)}{2}\right]}$$

The Atom-Bond Connectivity index[4] is given by

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_G(u) + d_G(v) - 2}{d_G(u)d_G(v)}}$$

The Sum Connectivity index[7] of a graph is

$$SCI(G) = \sum_{uv \in E(G)} \frac{2}{\sqrt{d_G(u) + d_G(v)}}$$

The domination degree of a vertex is given by $D_G(w) = |Y(G) - d_G(w)|$, where $Y(G)$ is the domination number. The above mentioned degree based topological indices can be redefined in terms of their domination degree as follows:

The Domination Harmonic index of a graph G is given by

$$DHI(G) = \sum_{uv \in E(G)} \frac{2}{D_G(u) + D_G(v)}.$$

The Dominational Geometric Arithmetic index of a graph is defined as

$$DGA(G) = \sum_{uv \in E(G)} \frac{\sqrt{D_G(u)D_G(v)}}{\left[\frac{D_G(u) + D_G(v)}{2}\right]}.$$

The Dominational Atom-Bond Connectivity index is given by

$$DABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{D_G(u) + D_G(v) - 2}{D_G(u)D_G(v)}}.$$

The Sum Connectivity index of a graph is

$$DSCI(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{D_G(u) + D_G(v)}}.$$

The newly defined topological index are generalized for a family of standard graphs and for the corona product of few graphs.

Generalization for standard graphs

Theorem 1. For any r -regular graph G with n vertices and $\frac{nr}{2}$ edges,

- $$DHI(G) = \frac{nr}{2(r-1)},$$

- $$DGA(G) = \frac{nr}{2},$$

- $$DABC(G) = \sqrt{\frac{r-2}{2}} \left(\frac{nr}{r-1}\right)$$

- $$DSCI(G) = \frac{nr}{2\sqrt{2(r-1)}}$$

Proof. If G is a r -regular graph with n nodes and $\frac{nr}{2}$ lines, then the domination number of G is r . $D_G(v_i) = r - 1, \forall v_i \in V(G)$.

The edges can be classified as:

$$E_\alpha = \{D_G(v_i) = D_G(v_j) = r - 1\}, |E_\alpha| = \frac{nr}{2}.$$

Hence,

$$DHI(G) = \frac{nr}{2(r-1)},$$

$$DGA(G) = \frac{nr}{2},$$

$$DABC(G) = \sqrt{\frac{r-2}{2}} \left(\frac{nr}{r-1}\right),$$

$$DSCI(G) = \frac{nr}{2\sqrt{2(r-1)}}.$$

Corollary 1 In G is a complete graph with n vertices then

$$DHI(G) = \frac{n(n-1)}{2(n-2)},$$

$$DGA(G) = \frac{n(n-1)}{2},$$

$$DABC(G) = \sqrt{\frac{n-3}{2} \frac{n(n-1)}{n-2}},$$

$$DSCI(G) = \frac{n(n-1)}{2\sqrt{2(n-2)}}.$$

Proof. A complete graph with n vertices is always $n - 1$ regular. Thus, $r = n - 1$ and hence the result.

Theorem 2. For any Path graph $P_n(n \geq 3)$

- $$DHI(G) = \frac{n-3}{|P_{Y-2}|} + \frac{4}{|P_{Y-1}|+|P_{Y-2}|},$$
- $$DGA(G) = (n-3) + 4 \frac{\sqrt{|P_{Y-1}||P_{Y-2}|}}{|P_{Y-1}|+|P_{Y-2}|},$$
- $$DABC(G) = (n-3) \frac{\sqrt{2|P_{Y-2}|-2}}{|P_{Y-2}|} + \sqrt{\frac{|P_{Y-1}|+|P_{Y-2}|-2}{|P_{Y-2}||P_{Y-1}|}},$$
- $$DSCI(G) = \frac{(n-3)}{\sqrt{2|P_{Y-2}|}} + \frac{2}{\sqrt{|P_{Y-1}|+|P_{Y-2}|}}$$

Proof: A path graph resembles a straight line with n nodes on it and $n - 1$ edges. For a path graph, $P_Y = \lceil \frac{n}{3} \rceil$. The dominational degrees are as follows:

$$E_1 = \{D_G(u) = D_G(v) = |P_Y - 2|\}, |E_1| = (n - 3)$$

$$E_2 = \{D_G(u) = |P_Y - 2|, D_G(v) = |P_Y - 1|\}, |E_2| = 2.$$

Thus, the result.

Theorem 3. For any Cycle graph $P_n(n \geq 3)$

- $$DHI(G) = \frac{n}{|C_Y-2|},$$
- $$DGA(G) = n,$$
- $$DABC(G) = n \frac{\sqrt{2|C_Y-2|-2}}{|C_Y-2|},$$
- $$DSCI(G) = n \sqrt{\frac{1}{|C_Y-2|}}.$$

Proof. A cycle is a 2-regular connected cyclic graphs with equal number of vertices and edges, i.e., $|V(G)| = |E(G)| = n$.

In a cycle graph, $C_Y = \lceil \frac{n}{3} \rceil$. The edges are split based on their dominational degree as follows:

$$E_1 = \{D_G(u) = D_G(v) = |C_Y - 2|\}, |E_1| = n$$

Thus,

$$DHI(G) = \frac{n}{|C_Y-2|},$$

$$DGA(G) = n,$$

$$DABC(G) = n \frac{\sqrt{2|C_Y-2|-2}}{|C_Y-2|},$$

$$DSCI(G) = n \sqrt{\frac{1}{|C_Y-2|}}.$$

Theorem 4. For a Dutch Windmill graph $D_n^{(m)}$,

- $$DHI(G) = \frac{4m}{|\alpha-2|+|\alpha-2m|} + \frac{m(n-2)}{|\alpha-2|},$$
- $$DGA(G) = m(n-2) + 4m \frac{\sqrt{|\alpha-2m||\alpha-2|}}{|\alpha-2m|+|\alpha-2|},$$
- $$DABC(G) = 2m \sqrt{\frac{|\alpha-2m|+|\alpha-2|-2}{|\alpha-2m||\alpha-2|}} + \frac{m(n-2)}{|\alpha-2|} \sqrt{2|\alpha-2|-2},$$
- $$DSCI(G) = \frac{2m}{\sqrt{|\alpha-2m|+|\alpha-2|}} + \frac{m(n-2)}{\sqrt{2|\alpha-2|}}.$$

Proof. A Dutch Windmill graph [8] is built by taking m replica of a cycle with n vertices. The order and size of D_n^m are $(n-1)m+1$ and mn respectively.

By algebraic method, the domination number of D_n^m is $\alpha = 1 + m \lfloor \frac{n-3}{3} \rfloor$.

Hence,

Sl No	Edge Partition	Number of edges
	$D_G(u) = \alpha - 2m , D_G(v) = \alpha - 2 $	$2m$
	$D_G(u) = \alpha - 2 , D_G(v) = \alpha - 2 $	$m(n - 2)$

Thus,

$$DHI(G) = \frac{4m}{|\alpha-2|+|\alpha-2m|} + \frac{m(n-2)}{|\alpha-2|},$$

$$DGA(G) = m(n-2) + 4m \frac{\sqrt{|\alpha-2m||\alpha-2|}}{|\alpha-2m|+|\alpha-2|},$$

$$DABC(G) = 2m \sqrt{\frac{|\alpha-2m|+|\alpha-2|-2}{|\alpha-2m||\alpha-2|}} + \frac{m(n-2)}{|\alpha-2|} \sqrt{2|\alpha-2|-2},$$

$$DSCI(G) = \frac{2m}{\sqrt{|\alpha-2m|+|\alpha-2|}} + \frac{m(n-2)}{\sqrt{2|\alpha-2|}}.$$

Theorem 5 For a Kulli Cycle Windmill graph $C_{n+1}^{(m)}$,

- $$DHI(G) = \frac{mn(mn+5)}{2(mn+1)},$$
- $$DGA(G) = mn \left[1 + \frac{\sqrt{2(mn-1)}}{mn+1} \right],$$
- $$DABC(G) = \sqrt{2}mn,$$
- $$DSCI(G) = mn \left[\frac{1}{2} + \frac{1}{\sqrt{mn+1}} \right].$$

Proof. A Kulli cycle windmill graph[9] is a combination of a complete graph with one vertex and m copies of a cycle with n vertices.

The total number of vertices is $mn + 1$, edges is $2mn$ and $Y(G) = 1$. The edge partition based on the the dominational degree is as follows:

$$E_1 = \{D_G(u) = D_G(v) = 2\}, |E_1| = mn$$

$$E_2 = \{D_G(u) = 2, D_G(v) = mn - 1\}, |E_2| = mn.$$

Hence, the result.

Theorem 6. For a Kulli Path Windmill graph $P_n^{(m)}$,

$$DHI(G) = \frac{4m}{3} + \frac{m(n-3)}{2} + \frac{4}{n} + \frac{2m(n-2)}{mn+1},$$

$$DGA(G) = \frac{\sqrt{2}}{3}4m + m(n-3) + \frac{4}{n}\sqrt{mn-1} + \frac{2m(n-2)}{mn+1}\sqrt{2(mn-1)},$$

$$DABC(G) = \sqrt{2}m + \frac{m}{\sqrt{2}}(n-3) + \sqrt{\frac{mn-2}{mn-1}}2m + \frac{m(n-2)}{\sqrt{2}},$$

$$DSCI(G) = \frac{2m}{\sqrt{3}} + \frac{(mn-3m)}{2} + \frac{2m}{\sqrt{mn}} + \frac{m}{\sqrt{mn+1}}(n-2).$$

Proof. A Kulli Path Windmill graph[10] is brought out by the union of m copies of path with n vertices and K_1 at the centre.

The order and size of such graphs are $mn + 1$ and $2mn - m$, with domination number equal to unity. The partition of edges is shown in the table below.

Sl No	Edge Partition	Number of edges
	$D_G(u) = 1, D_G(v) = 2$	$2m$
	$D_G(u) = 2, D_G(v) = 2$	$mn - 3m$
	$D_G(u) = mn - 1, D_G(v) = 1$	$2m$
	$D_G(u) = mn - 1, D_G(v) = 2$	$mn - 2m$

Hence, the result.

RESULTS ON CORONA PRODUCT OF GRAPHS

The Corona product[11] of two graphs $G = (p, q)$ and $H = (p', q')$ is denoted by $G \circ H$, is obtained by joining every vertex of G with a copy of H . As a consequence, the corona product has order $p + p'$ and size $(q' + p')p + q$.

Theorem 7 For the corona product of $K_n (n \geq 1) \circ P_m (m \geq 2)$,

$$DHI(G) = \frac{4n}{|n-2|+|n-3|} + \frac{n(m-3)}{|n-3|}$$

$$DGA(G) = n(m-3) + 4n \frac{\sqrt{|n-2||n-3|}}{|n-2|+|n-3|},$$

$$DABC(G) = 2n \sqrt{\frac{|n-3|+|n-2|-2}{|n-3||n-2|}} + n(m-3) \frac{\sqrt{2|n-3|-2}}{|n-3|},$$

$$DSCI(G) = \frac{2m}{\sqrt{3}} + \frac{(mn-3m)}{2} + \frac{2m}{\sqrt{mn}} + \frac{m}{\sqrt{mn+1}}(n-2).$$

Proof. The Corona product of $K_n (n \geq 1) \circ P_m (m \geq 2)$ has the following order and size.

Graph	Vertices	Edges
K_n	n	$\frac{n(n-1)}{2}$
n copies of P_m	mn	$(m-1)n$
Each vertex of K_n with P_m	-	mn
Total	$(m+1)n$	$\frac{n(n-1)}{2} + 2mn - n$

The domination number of the graph is n with edge partition as follows:

$$E_1 = \{D_G(u) = |n-2|, D_G(v) = |n-3|\}, |E_1| = 2n,$$

$$E_2 = \{D_G(u) = |n-3| = D_G(v)\}, |E_2| = (m-3)n.$$

Hence,

$$DHI(G) = \frac{4n}{|n-2|+|n-3|} + \frac{n(m-3)}{|n-3|}$$

$$DGA(G) = n(m-3) + 4n \frac{\sqrt{|n-2||n-3|}}{|n-2|+|n-3|}$$

$$DABC(G) = 2n \sqrt{\frac{|n-3|+|n-2|-2}{|n-3||n-2|}} + n(m-3) \frac{\sqrt{2|n-3|-2}}{|n-3|}$$

$$DSCI(G) = \frac{2m}{\sqrt{3}} + \frac{(mn-3m)}{2} + \frac{2m}{\sqrt{mn}} + \frac{m}{\sqrt{mn+1}}(n-2).$$

On the same basis, we can obtain the following results.

1. For the corona product of $K_n (n \geq 1) \circ C_m (m \geq 3)$,

-
$$DHI(G) = \frac{mn}{|n-3|}$$

-
$$DGA(G) = mn,$$

-
$$DABC(G) = mn \frac{\sqrt{2|n-3|-2}}{n-3},$$

-
$$DSCI(G) = mn \sqrt{\frac{2}{|n-3|}}.$$

2. For the corona product of $K_p (p \geq 1) \circ K_{m,n}$,

-
$$DHI(G) = \frac{2mnp}{|p-m|+|p-n|}$$

-
$$DGA(G) = \sqrt{|p-m||p-n|} DHI(G),$$

-
$$DABC(G) = mnp \sqrt{\frac{|p-m|+|p-n|-2}{|p-m||p-n|}},$$

-
$$DSCI(G) = \frac{2mnp}{\sqrt{|p-m|+|p-n|}}.$$

CONCLUSION

Although topological indices are called molecular descriptors, they are altered or redefined and utilised to determine any physiochemical feature of a substance or to analyse the activity of a specific enzyme. In graphs, the dominant set of vertices rule the entire graph, similar to how the brain dominates the nervous system and sensory organs. This paper presents a research on dominant rheological descriptors, which lays the path for novel applications based on these descriptors.

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